Meeting 2 Report – The University of Manchester

Author: Stephen Longshaw (STFC)

The second UK Fluids Network Smoothed Particle Hydrodynamics (SPH) Special Interest Group (SIG) was hosted by the University of Manchester's school of MACE in the George Begg Building on the 12th of April 2018.

SPH SIG Members in front of the original Reynolds experiment at the University of Manchester.

The following SIG members were present at the meeting (by organisation):

STFC: Xiaohu Guo (XG); Stephen Longshaw (SML); Alex Skillen (AS); Chrysovalantis Tsinginos (CT)
The University of Manchester: Annelie Baines (AB); Alex Chow (AC); Aaron English (AE); Steven Lind (SL); Benedict Rogers (BR); Peter Stansby (PS); Abouzied Nasar (AN); Jack King (JK); Katy Walton (KW); Georgina Reece (GR); Janwar Nurdin (JN); Hafiz Aslami (HA); Joseph O'Conner (JC)
Digital Engineering & Test Centre: Samuele De Guido (SDG)
Brunel University London: James Campbell (JC)
The University of Bristol: Thomas Rendall (TR); Samantha Huntley (SH)
Wilde Analysis Ltd.: Antonios Xenakis (AX)

University of Central Lancashire: Ahmed Wael Al Shear (AWAS)
The University of Birmingham: Alessio Alexiadis (AA)
The University of Dundee: Matthias Mimault (MM)
Bournemouth University: Richard Southern (RS); Valentin Miu (VM)
The University of Oxford: Martin Robinson (MR)
Swansea University: Min Lou (ML)
Others: Stephen Victory (SV)

The meeting saw an agenda that included eight technical presentations given around various SPH application themes, followed by discussion around a number of key topics important to the future of the SIG, including the next meeting that will target industry as well as how the SPH SIG can engage with industry better. A synopsis of this discussion is provided in section 2 of this document which is then followed by a number of the slide decks that were presented at the SIG.

1.0 Technical Presentations

Theme 1: Manufacturing and Industrial

Ahmad Wael Al Shaer: *SPH for laser welding*
James Campbell: *SPH for manufacturing and fracture*

Theme 2: Biological

Matthias Mimault: *An SPH model of the root tissue mechanics in the meristem*
Alessio Alexiadis: *A primer on Discrete Multiphysics with applications to biological systems*

Theme 3: Computing and Graphics

Richard Southern: *SPH sampling and its applications in computer graphics*
Xiaohu Guo: *SPH and High Performance Computing*

Theme 4: Geophysical and Environmental

Antonios Xenakis: *Landslides and tsunamis*
Benedict Rogers (on behalf of Georgios Fourtakas): *A multi-phase SPH model for simulating erosion and scouring using a critical bed-mobility condition and non-Newtonian models*

Dr James Campbell from Brunel University London presenting work on using SPH for manufacturing and fracture analysis

Dr Richard Southern from Bournemouth University talking about using SPH for sampling methods used in Computer Graphics

2.0 Group SIG Discussion

Note: For details of individuals named, please refer their abbreviation to the start of this document where each is defined.

Key points of discussion for the SIG at this meeting where:

1. How to better engage with communities outside the SIG.

There was a general consensus that this can be achieved through the UK Fluids Network website. SML made the point that all results hosted on the site are guaranteed for 5 years following September 2019, the challenge is how to provide original content that has wide-reaching impact while maintaining people's copyright on anything they may have already published either through their institution or journals etc.

There was a general agreement that the SIG would try and better engage with the Fluids Network public outlets. Our own Twitter account is already well managed by *AE*. A number if SIG members expressed interest in taking this forward. SML and BR agreed to look at how to better format the SPH SIG area of the site in the coming months.

2. The format and purpose of the next meeting in October 2018.

The next meeting will be hosted by the Digital Engineering Test Centre (DETC) at their London campus in the final week of October 2018. It was agreed that the scope of the meeting should remain industry focused. This means that SIG members who agree to take part will be part of a schedule designed to impart and enthuse novice industrial users to SPH as a method worth considering within their organisations. PS commented that this was not only important but essential for the SPH community in the UK.

There was discussion whether this should be a 1 day or 2-day event. Logistically 2 days makes more sense but as SDG pointed out, it is harder to engage with industry for 2 days due to overheads, especially given the London location. This will be decided as planning progresses.

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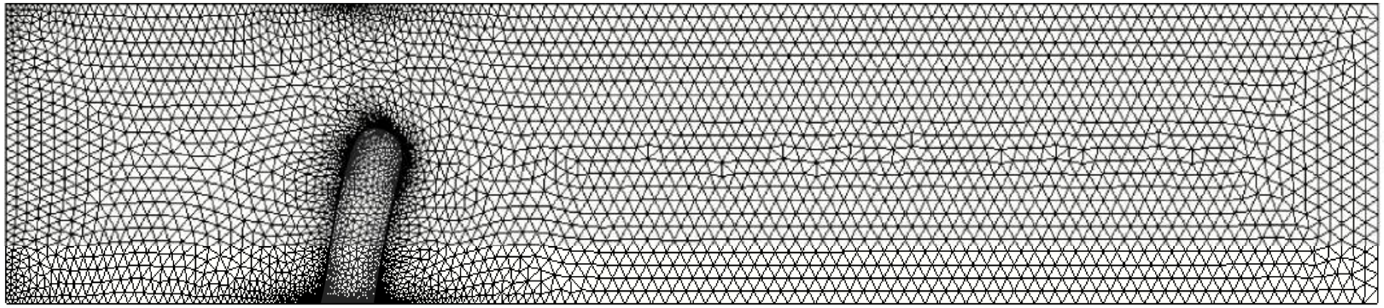
Alessio Alexiadis

School of Chemical Engineering

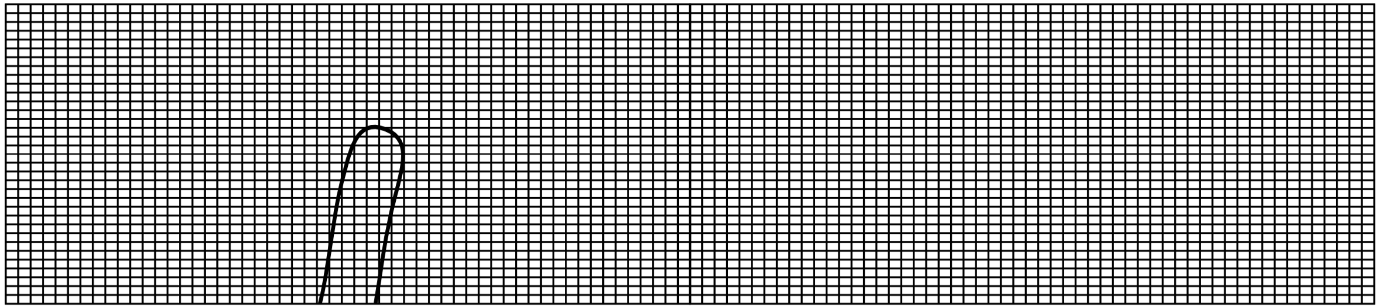
A primer on Discrete Multiphysics with applications to biological systems

SPH SIG MEETING, 12 APRIL 2018, MANCHESTER

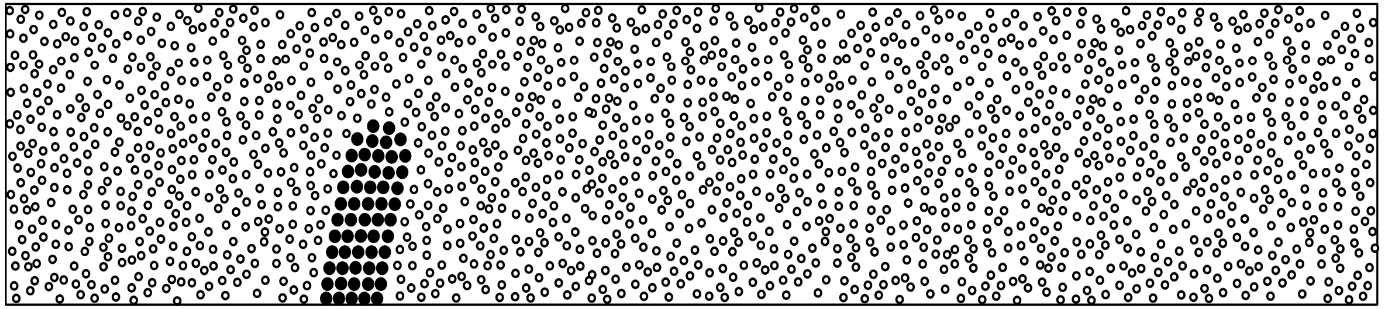
Discrete Multiphysics



(a) Conforming mesh



(b) Non-conforming mesh



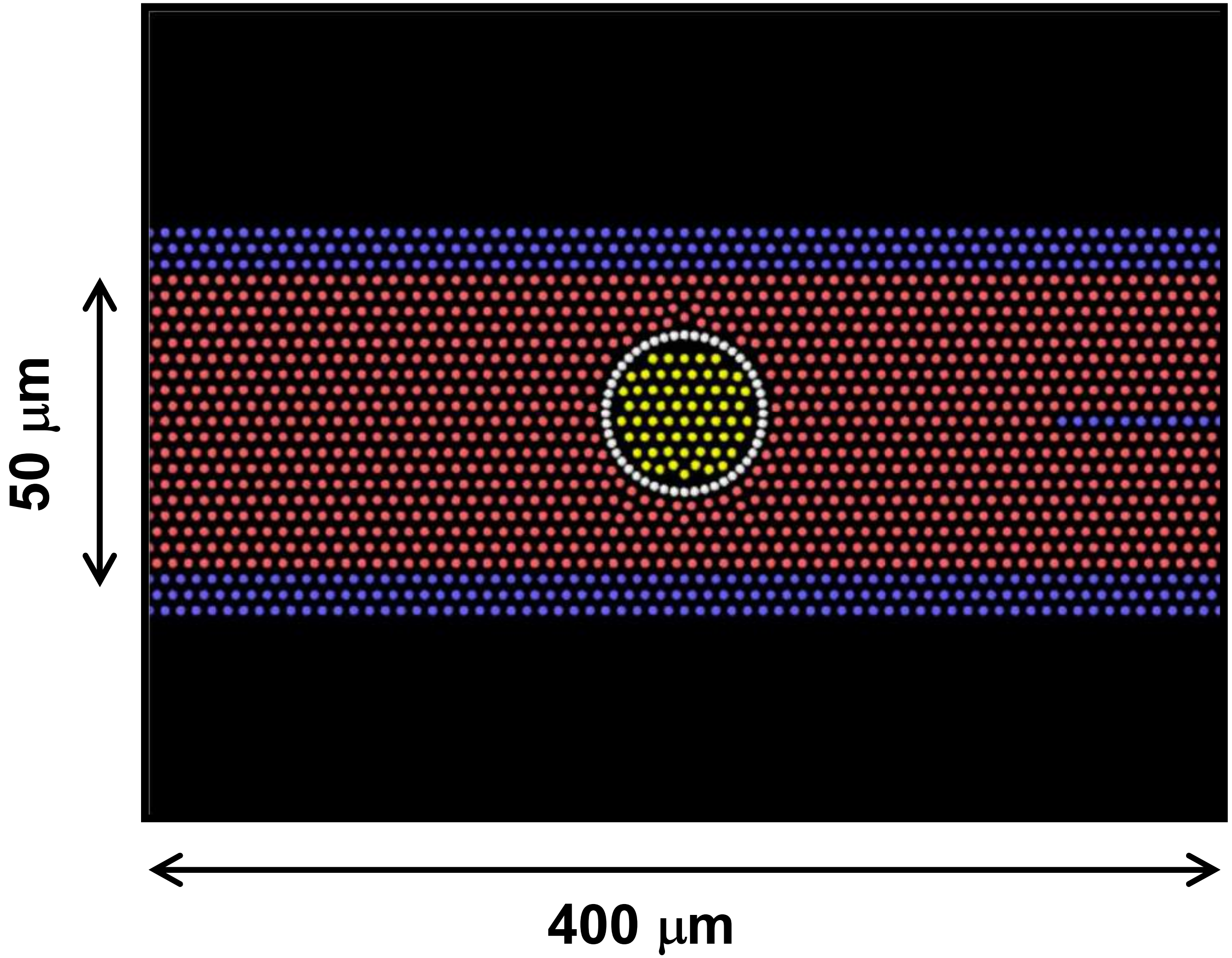
(c) Mesh-free

A “flipped” presentation

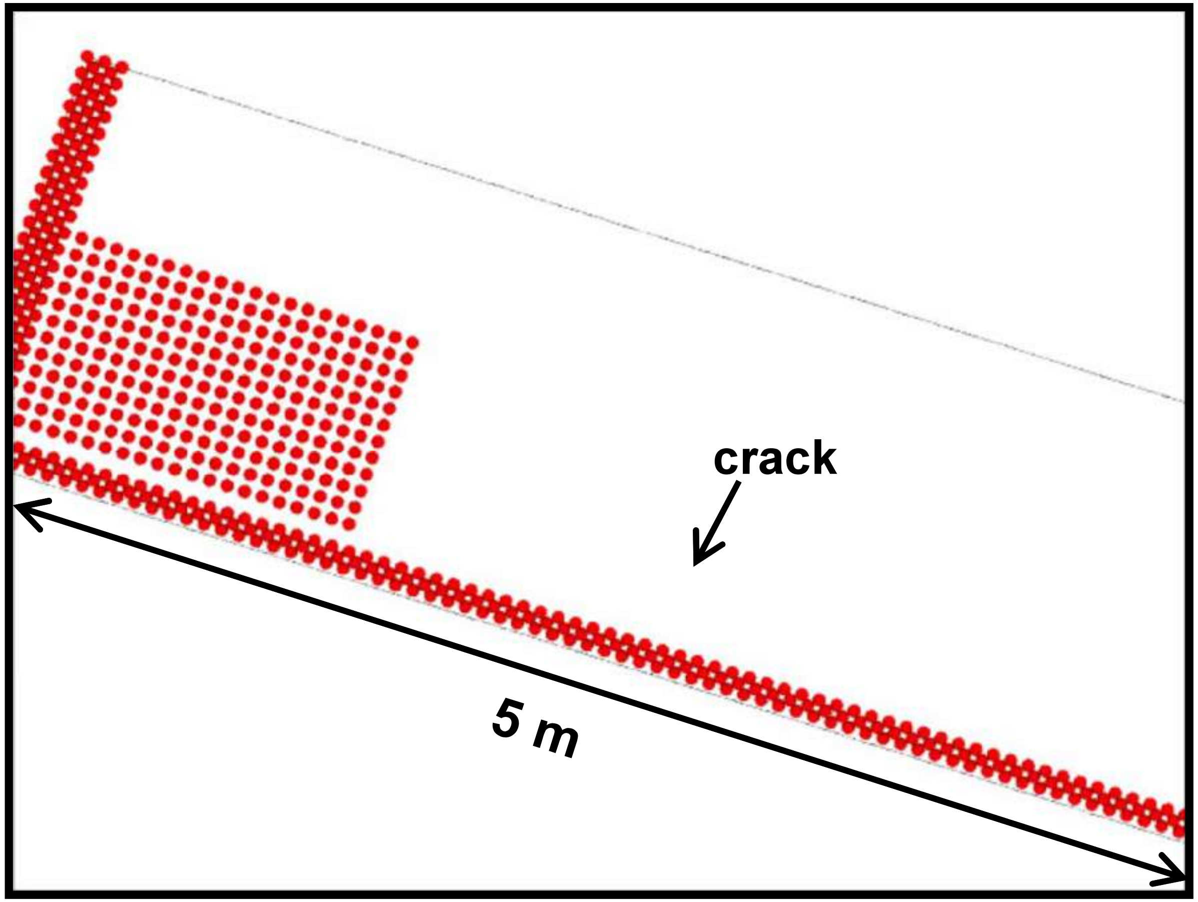
- Toy models
- (A brief) theoretical background
- Examples and applications

Toy models

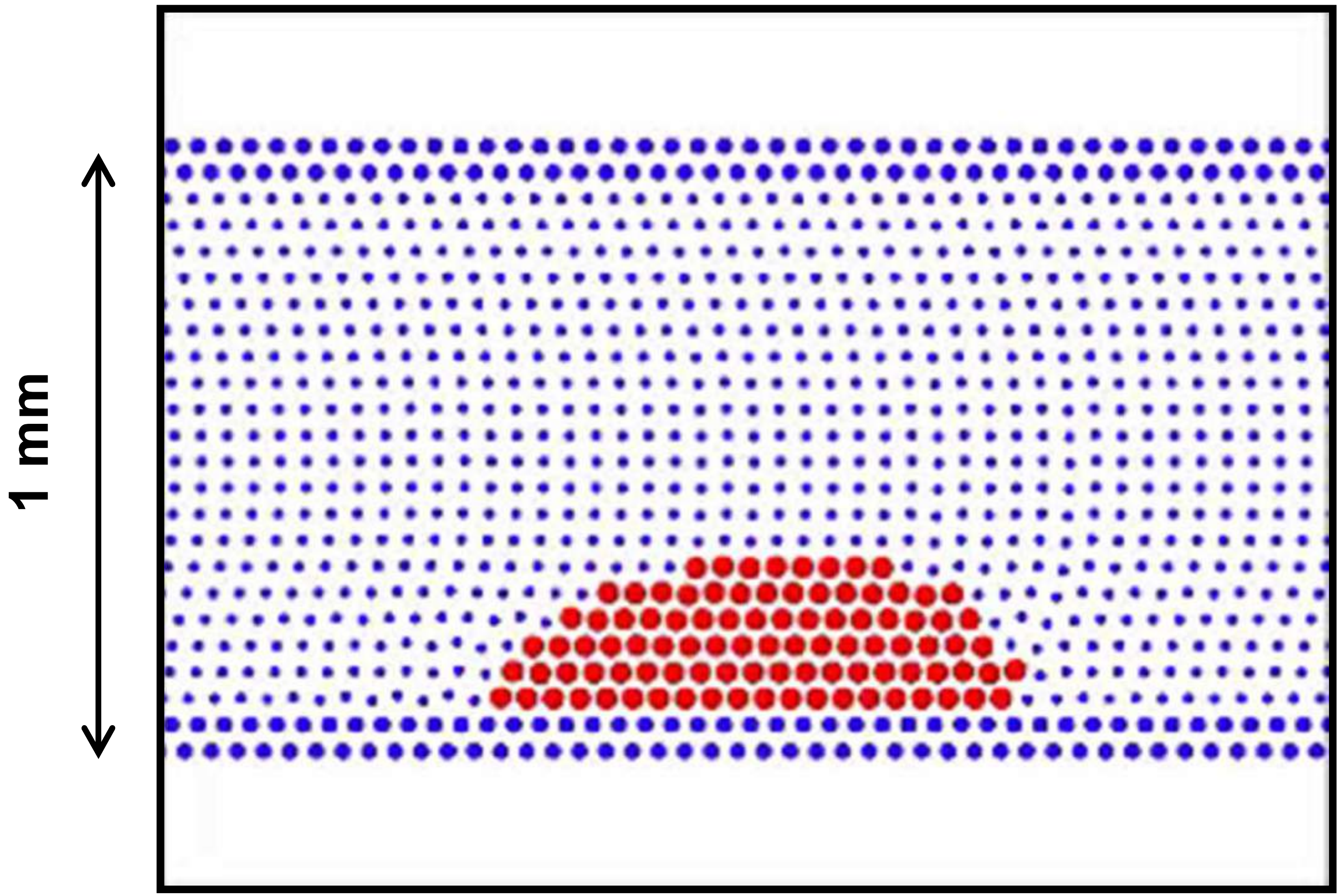
Cell break-up



Lava flow (solidification/Melting/breakage)



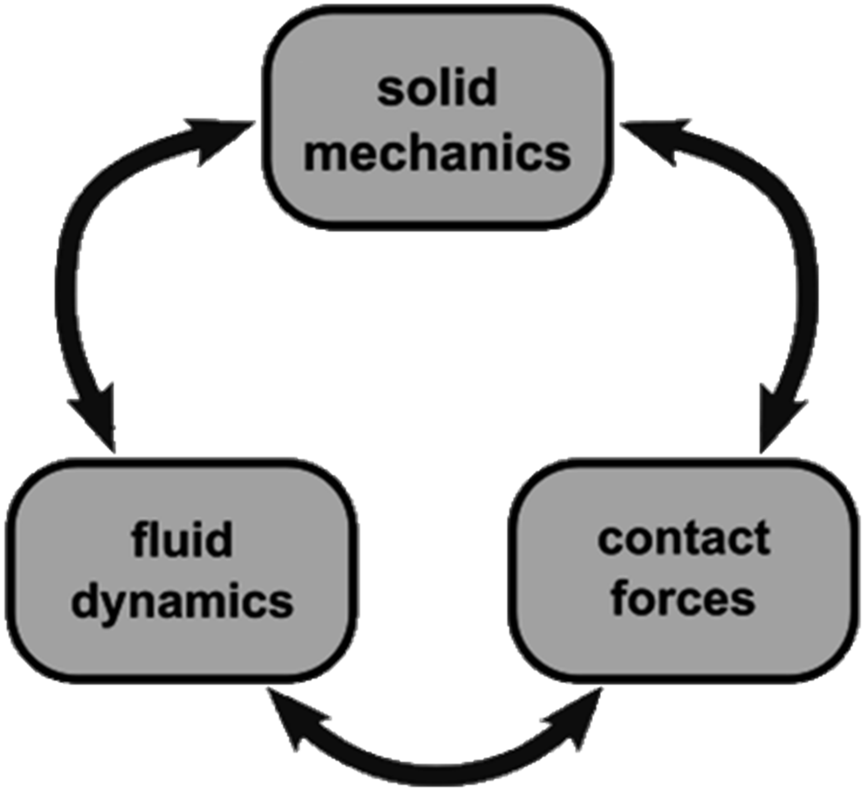
Cleaning process (fuzzy boundaries)



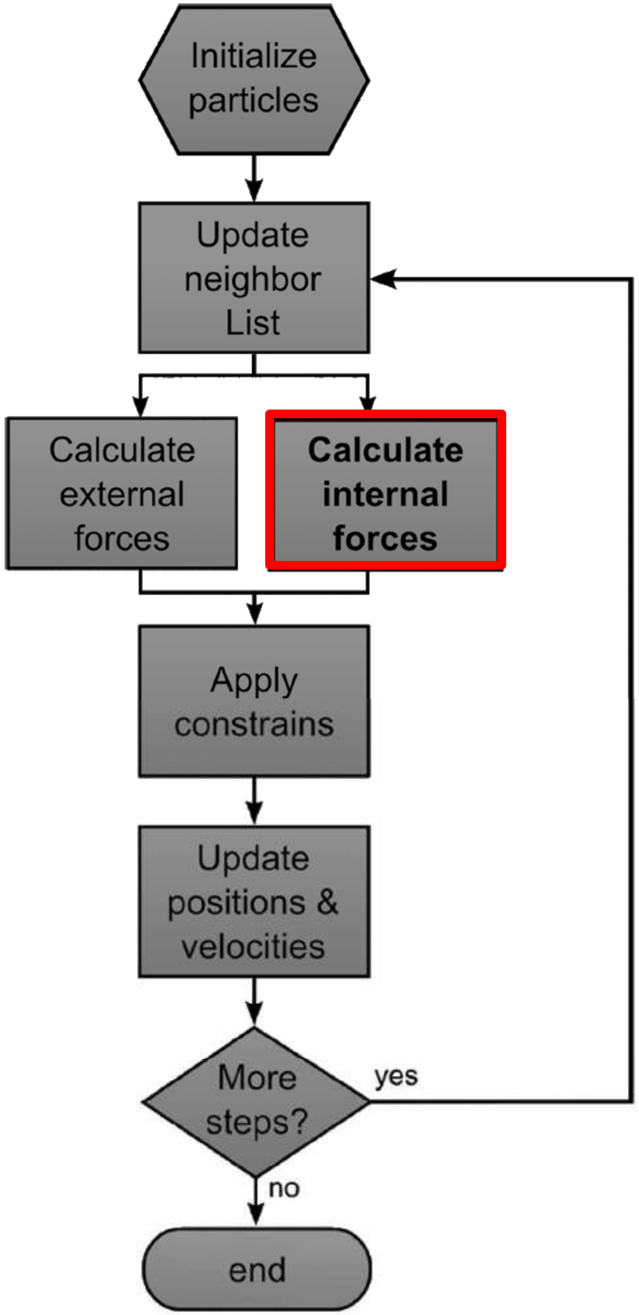
(A brief) theoretical background

The problem: modeling of solid-liquid flows and fluid-structure interactions

Different models for different phenomena; it is a Multiphysics problem



No model, alone, can describe the whole dynamics and we need a 'hybrid' approach

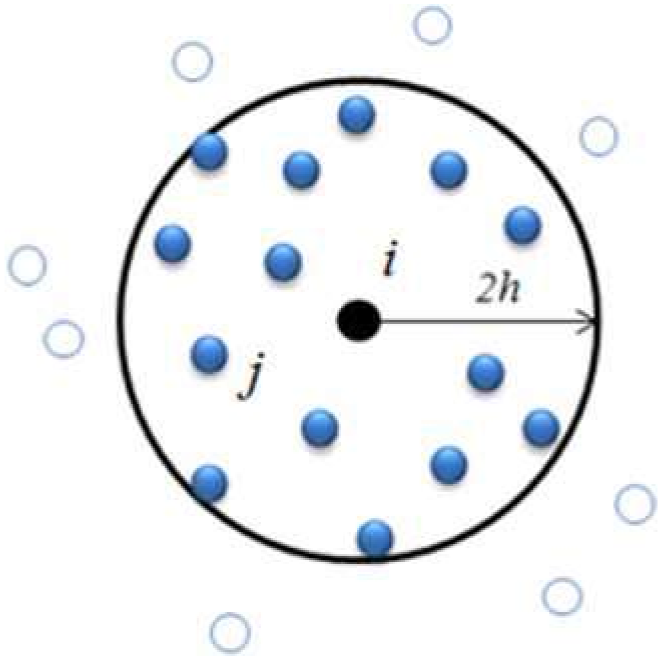


Not any mesh-free model though…

Our approach is based on

- **Smoothed Particle Hydrodynamics (SPH) for the liquid**
- **Lattice spring models (LDM) for the internal structure of the solid**
- **Discrete Element Method (DEM) for particle-particle interaction**

Smoothed Particle Hydrodynamics (liquids)

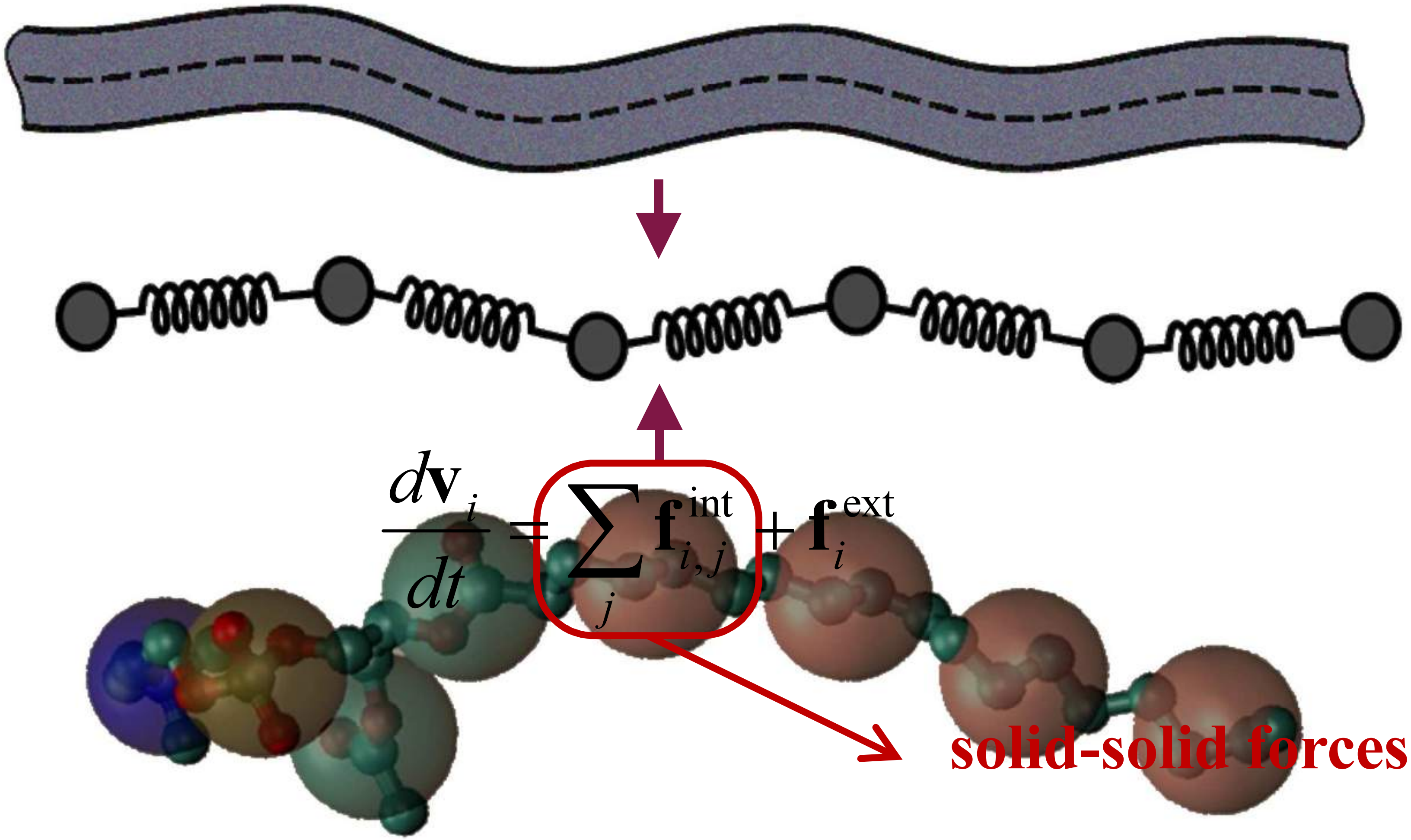


Conservation of momentum

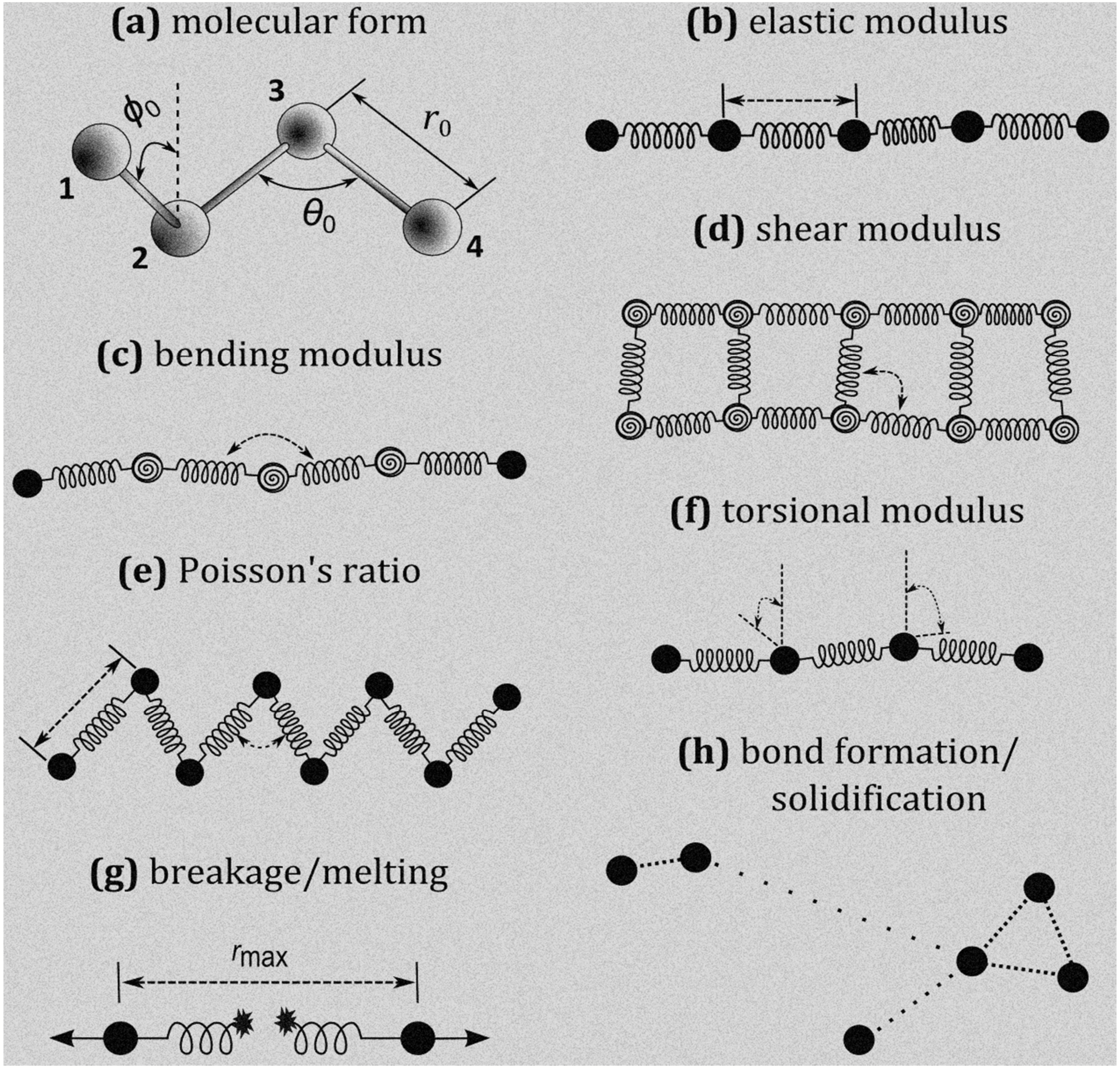
$$\frac{d\mathbf{v}_i}{dt} = \boxed{\sum_j m_i \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} + \Pi_{i,j} \right) \nabla_j W(\mathbf{r}_i - \mathbf{r}_j)} + \mathbf{f}_i^{\text{ext}}$$

liquid-liquid forces

Lattice Spring Model (solids)



$$\frac{d\mathbf{v}_i}{dt} = \boxed{\sum_j \mathbf{f}_{i,j}^{\mathrm{int}}} + \mathbf{f}_i^{\mathrm{ext}}$$



(a) molecular form
ϕ_0
1
2
3
4
r_0
θ_0
(b) elastic modulus
(c) bending modulus
(d) shear modulus
(e) Poisson's ratio
(f) torsional modulus
(g) breakage/melting
r_{max}
(h) bond formation/
solidification

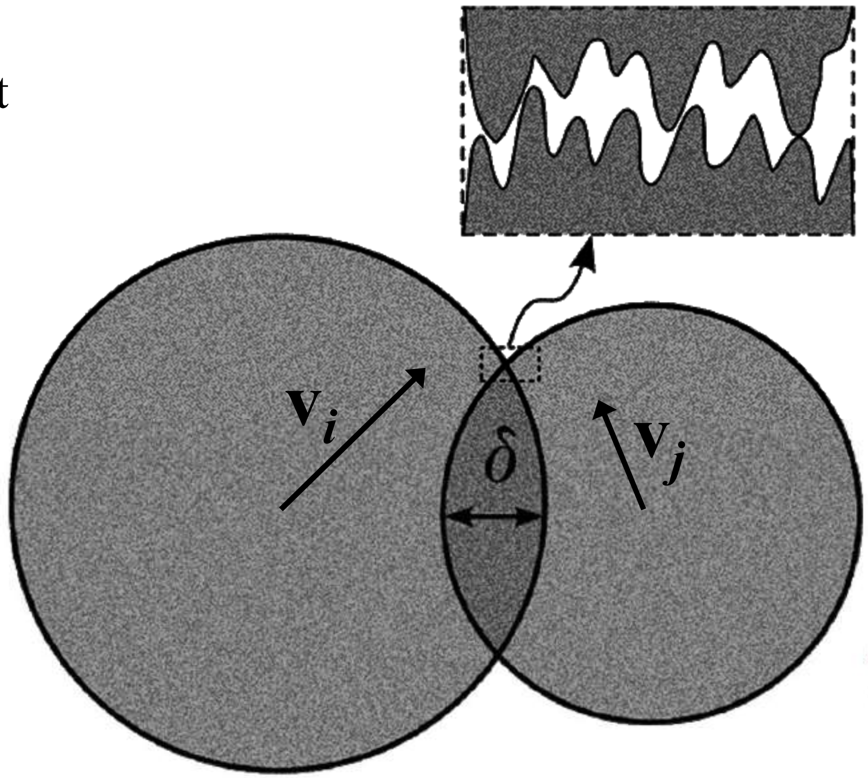
Discrete element method (DEM)

$$\frac{d\mathbf{v}_i}{dt} = \boxed{\sum_j \mathbf{f}^n_{i,j} + \sum_j \mathbf{f}^t_{i,j}} + \mathbf{f}_i^{\text{ext}}$$

normal and tangential contact forces (Hertzian model)

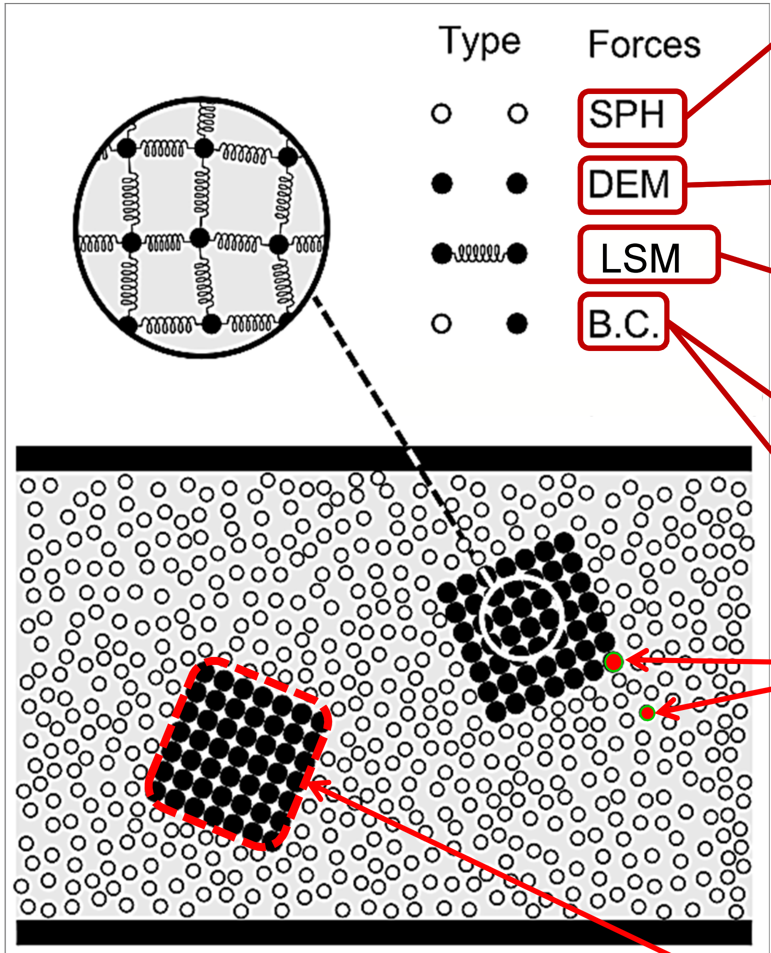
$$\mathbf{f}^n_{i,j} = \sqrt{\delta}\sqrt{\frac{R_i R_j}{R_i + R_j}}\left(k_n \delta \mathbf{n}_{i,j} - \gamma_n \mathbf{v}_n\right)$$

$$\mathbf{f}^n_{i,j} = -\sqrt{\delta}\sqrt{\frac{R_i R_j}{R_i + R_j}}\left(k_t \mathbf{\Delta s}_t - \gamma_t \mathbf{v}_t\right)$$



summarizing…

$$\frac{d\mathbf{v}_i}{dt} = \boxed{\sum_j \mathbf{f}_{i,j}^{\text{int}}} + \mathbf{f}_i^{\text{ext}}$$



$$\sum_j m_i m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} + \Pi_{i,j} \right) \nabla_j W_{i,j}$$

$$\begin{cases} f^n = \sqrt{\delta} R_{eff} \left(k_n \delta \mathbf{n}_{i,j} + \gamma_n \mathbf{v}_n \right) \\ f^t = -\sqrt{\delta} R_{eff} \left(k_t \Delta \mathbf{s}_t - \gamma_t \mathbf{v}_t \right) \end{cases}$$

$$\mathbf{F}_{i,j} = -k \left(r_0 - r_{i,j} \right)$$

computational particles

$$\left(\frac{\partial}{\partial t} u - v \right) \cdot n = 0$$

$$(\text{no} - \text{penetration})$$

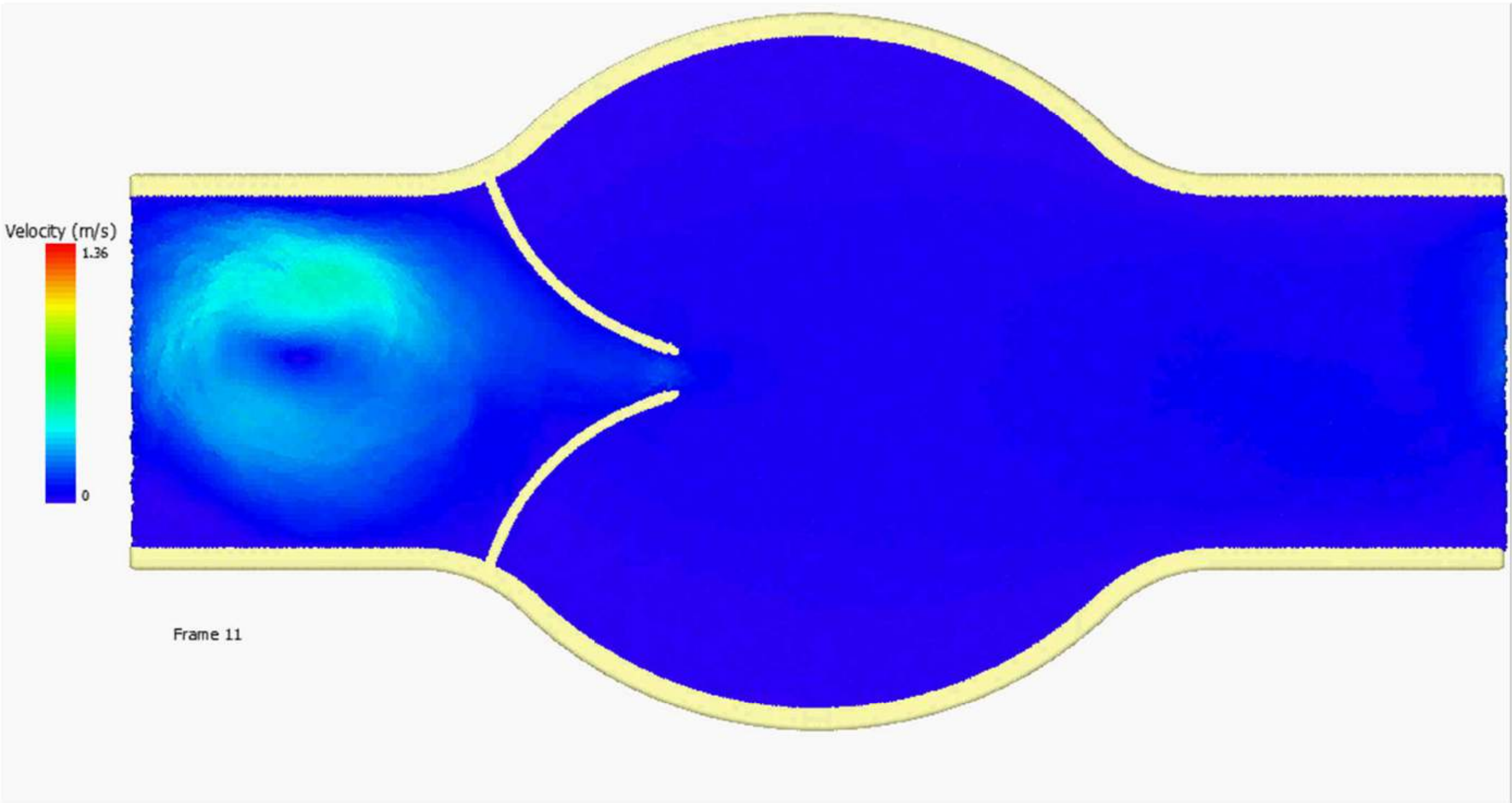
$$\left(\frac{\partial}{\partial t} u - v \right) \times n = 0$$

real particles

$$(\text{no} - \text{slip})$$

Examples and applications

Hearth Valves



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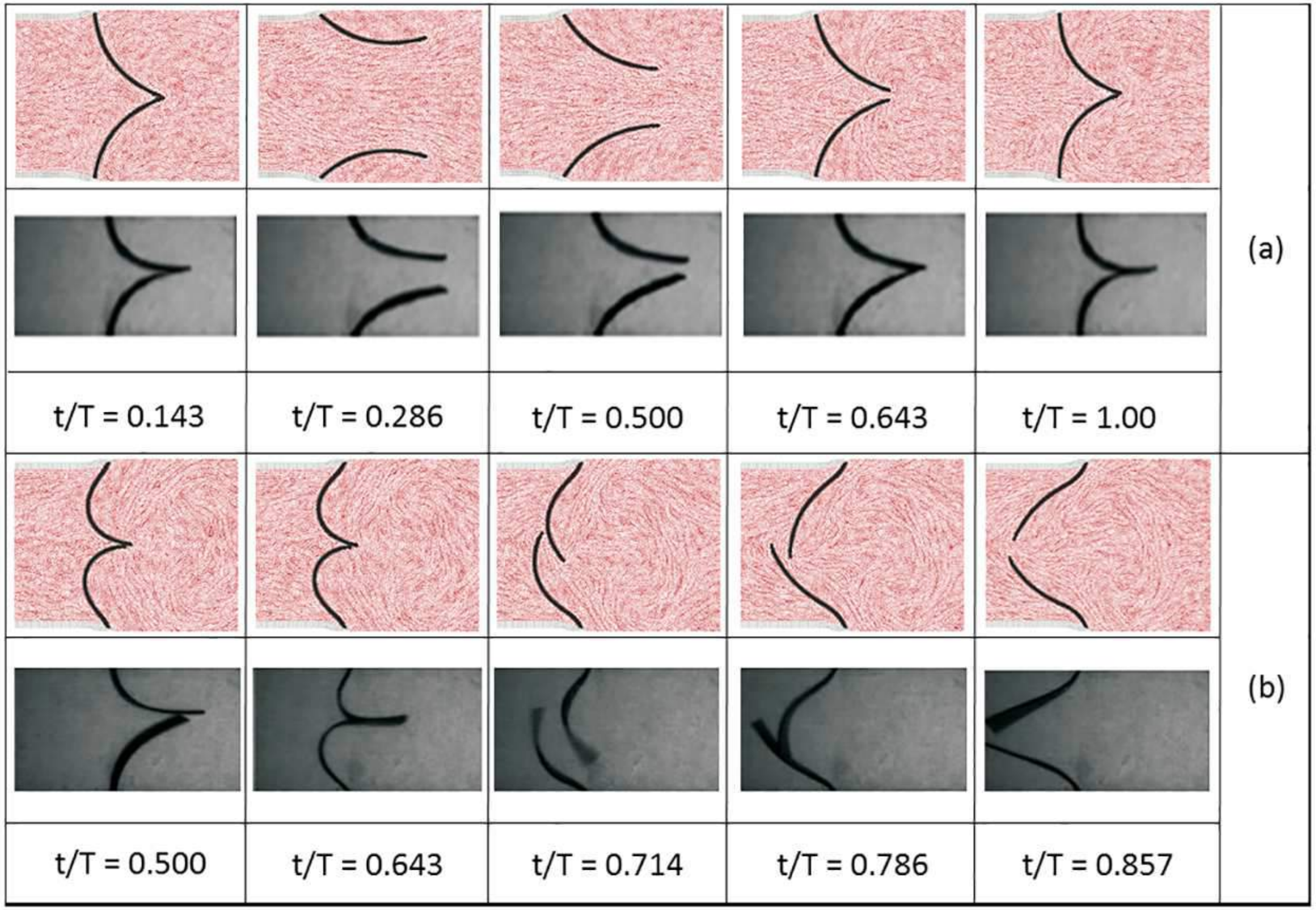
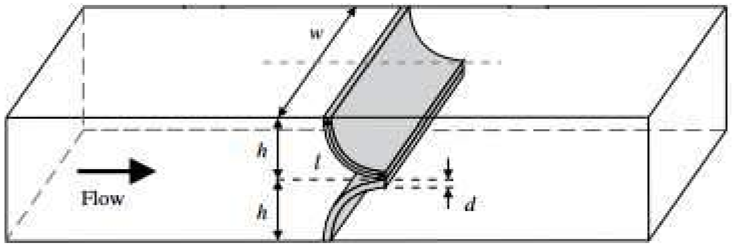
Leaflets dynamics





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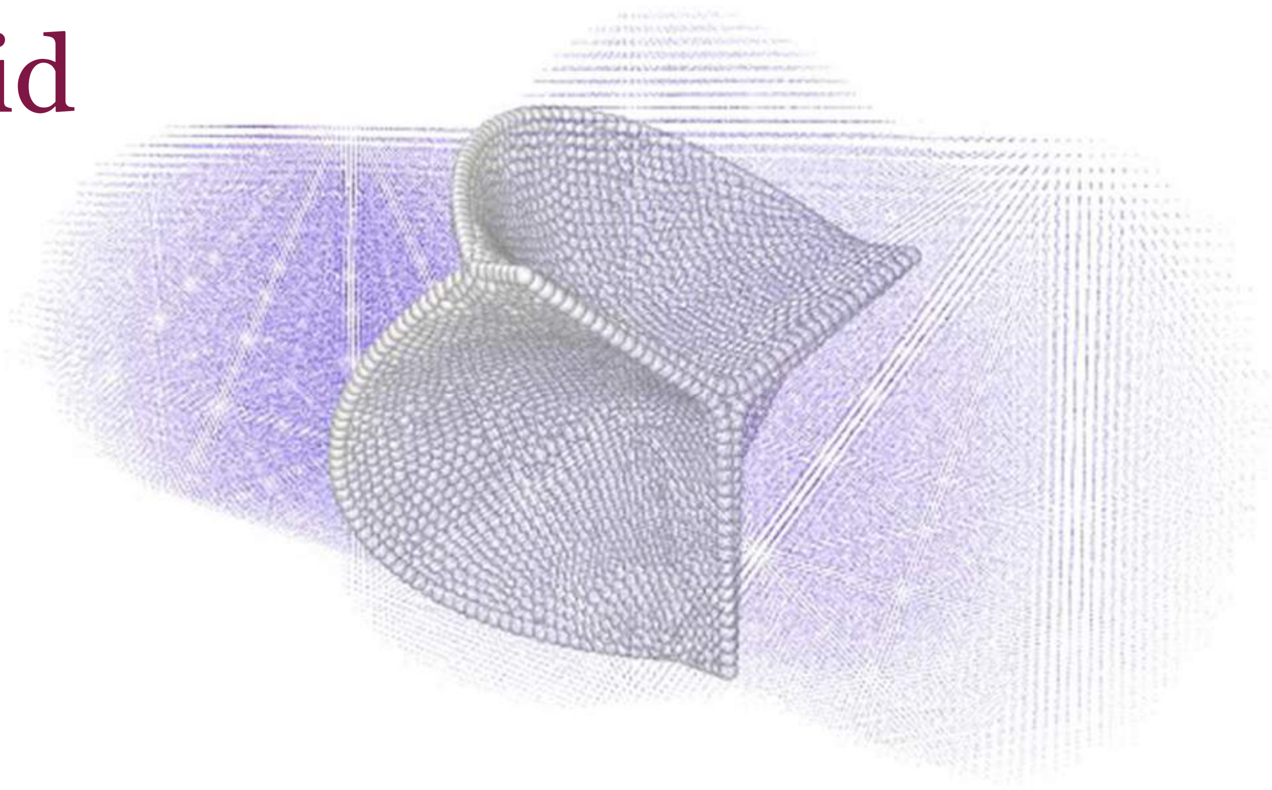
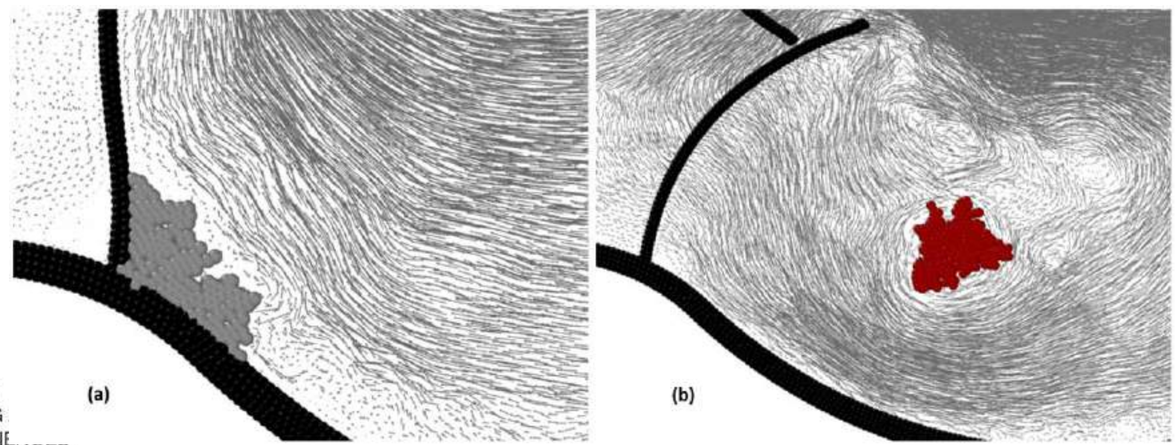
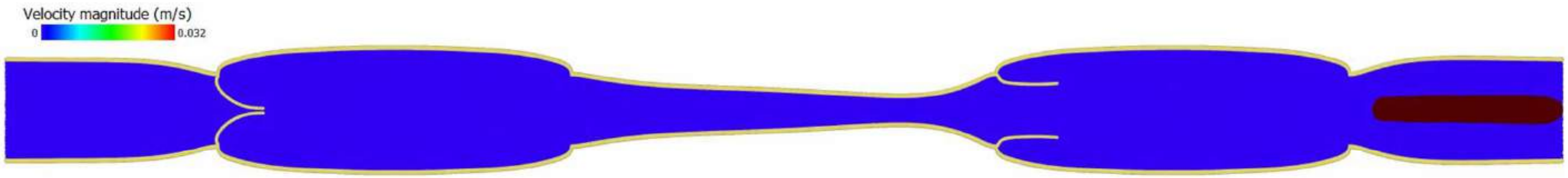
Ledesma-Alonso et al. J. Fluid Mech. (2014) **739**:338–362.

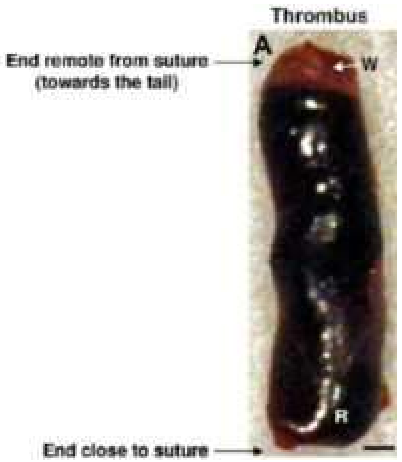
Tricuspid valves

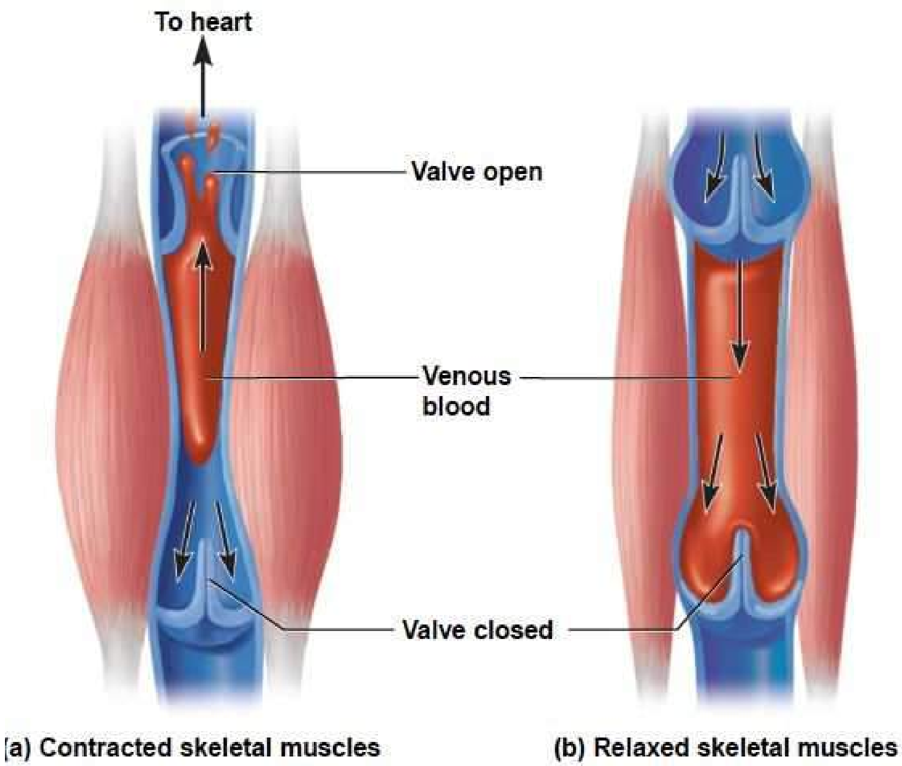


Clotting algorithm









Deep Venous Thrombosis

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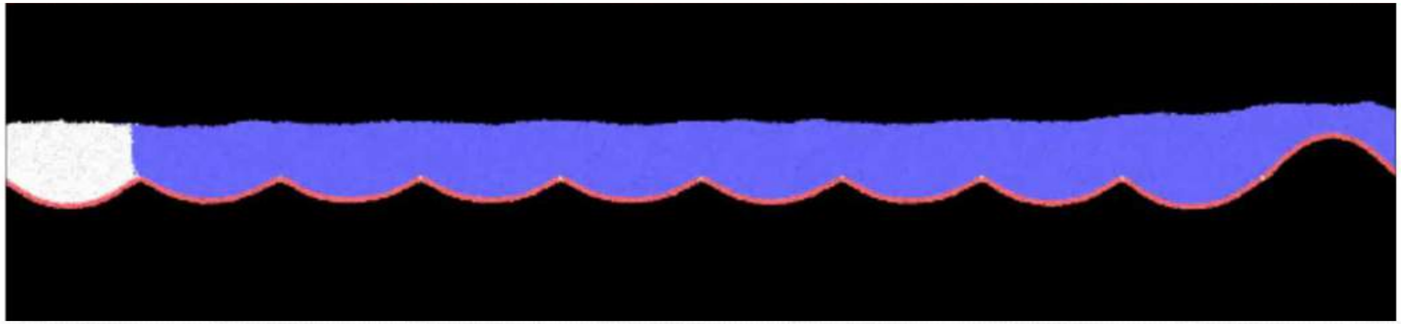
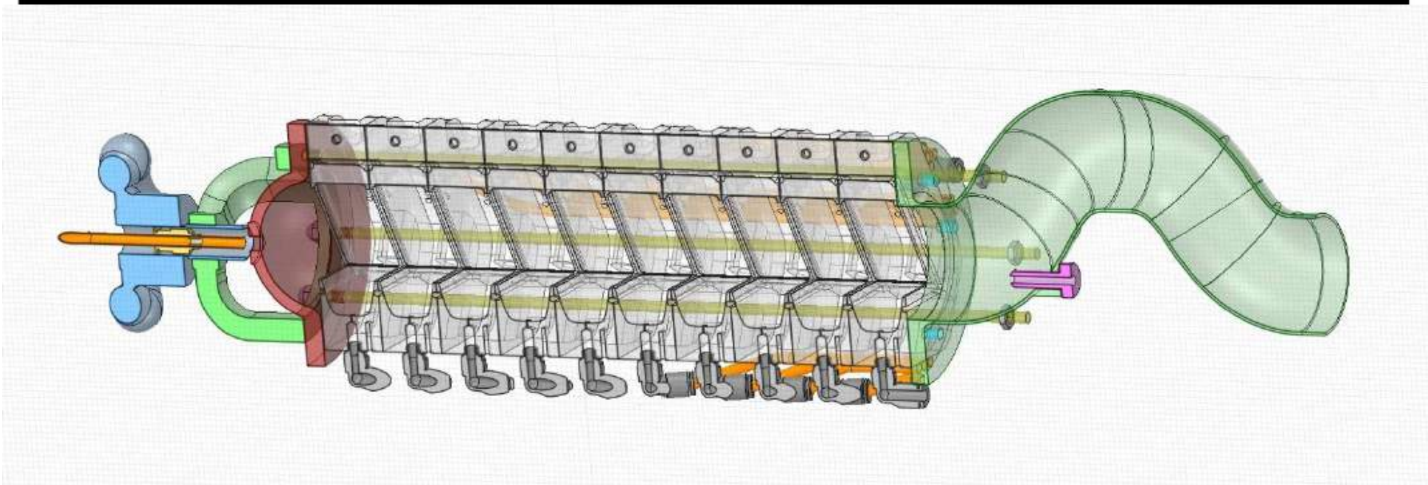
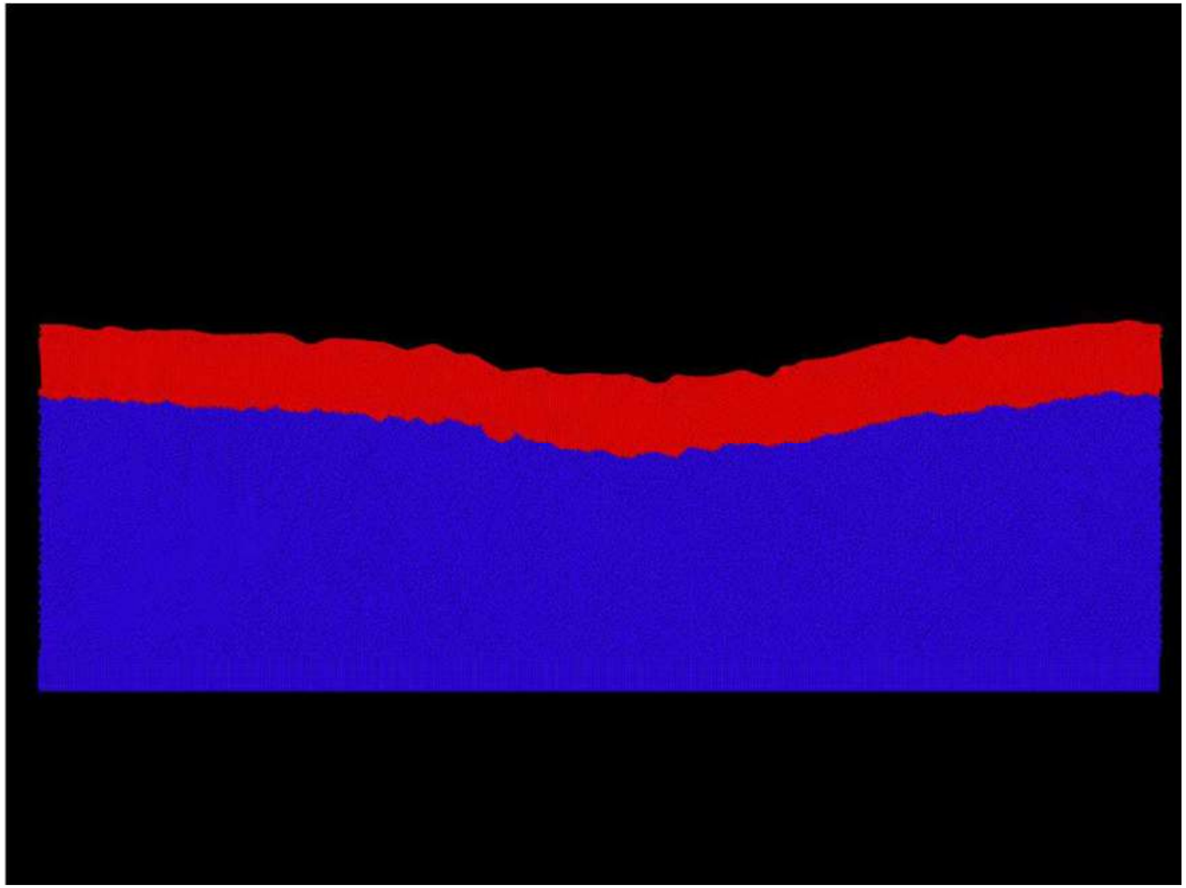
Intestine peristalsis

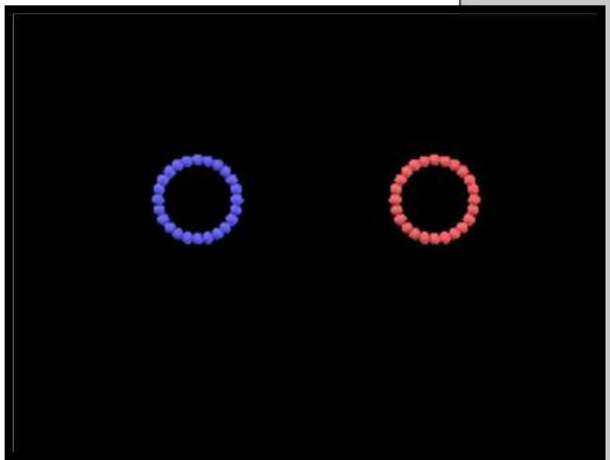


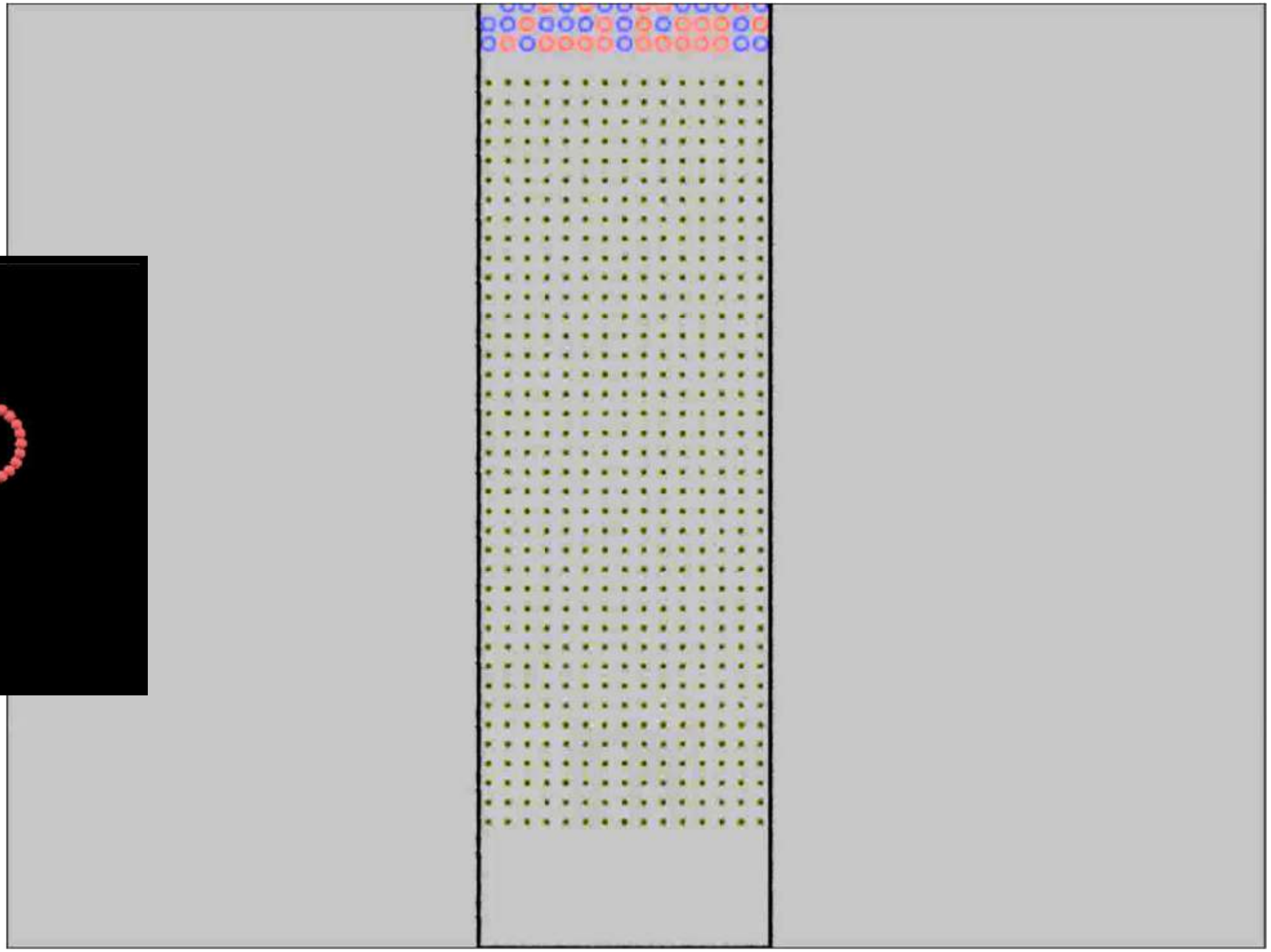


Mass transfer in lungs



Cells Separations (CGMD+DEM)





Current research

- Slurries
- Dense Emulsions
- Lava flows
- Cavitation Erosion
- Cleaning
- Cell separation
- Medical applications
- Biofilms

Acknowledgements

Horizon 2020 Programme

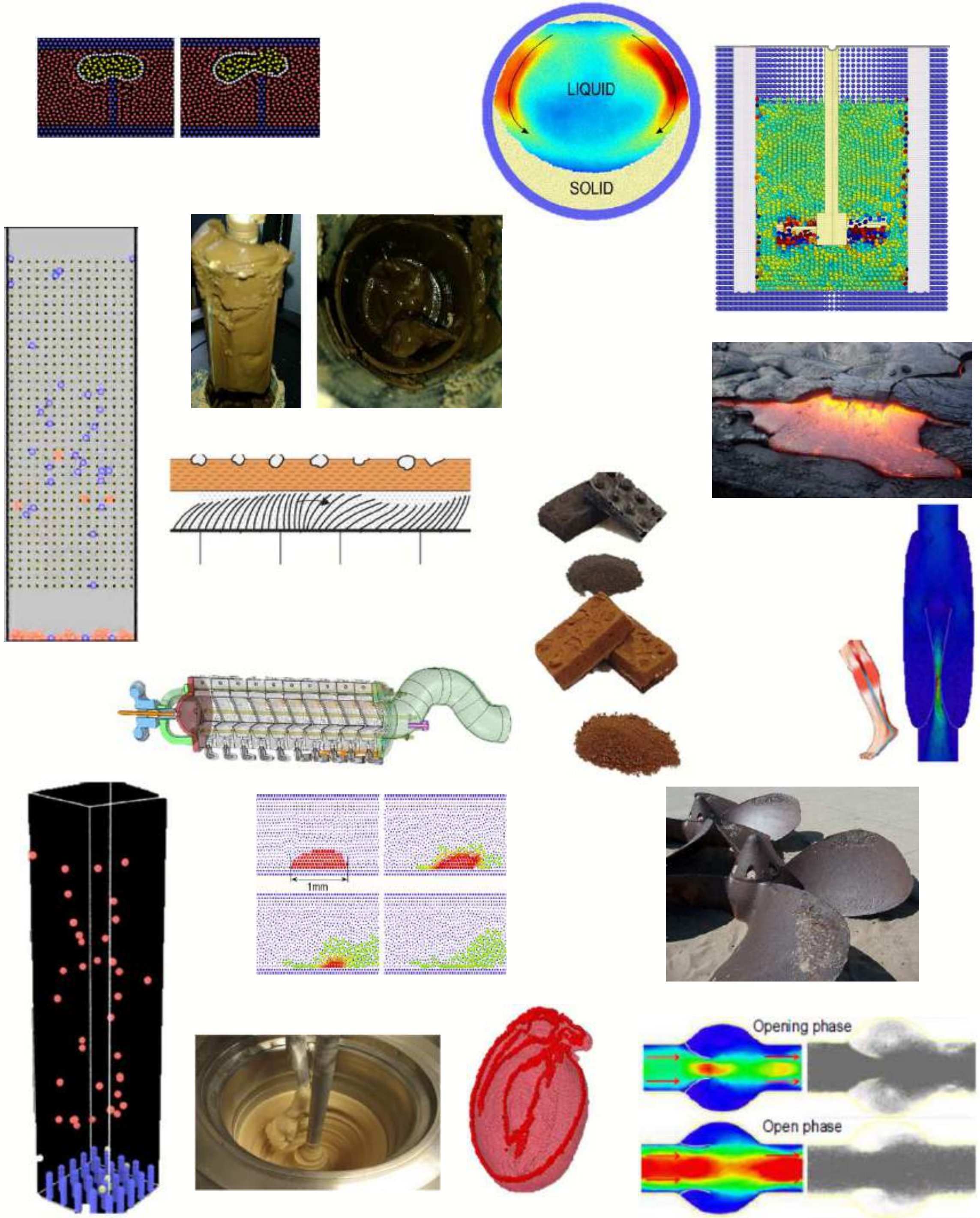
JM Johnson Matthey

EPSRC Engineering and Physical Sciences Research Council

Department of the Navy ONR Science & Technology

HPC Midlands Supercomputing for Research and Industry

ARC Australian Research Council



SPH for Manufacture and Fracture

James Campbell, Rade Vignjevic,

Kevin Hughes, Nenad Djordjevic,

Tom De Vuyst

Acknowledgement

- The EXTREME project leading to this presentation has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 636549.

- The EXTREME project aims at development novel material characterisation and in-situ measurement methods, **material models** and **improvement of computational methods** for the design and manufacture aerospace composite structures under EXTREME dynamic loadings leading to a significant reduction of weight, design and certification cost.

Brunel SPH research

Research group expertise:

- Solid and structural mechanics
- Crash, impact, shock response of materials and structures
- Constitutive modelling

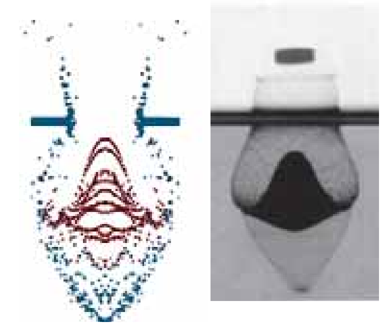
In-house developed SPH solver:

- Focussed on solid mechanics and fluid-structure interaction problems
- Coupled to 3D FEM solver
- Parallelised

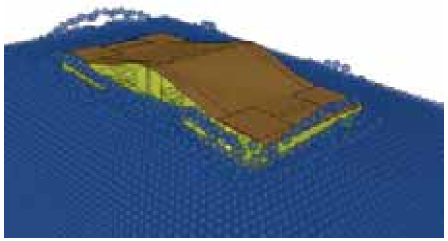
Applications:

- Hypervelocity Impact (original motivation)

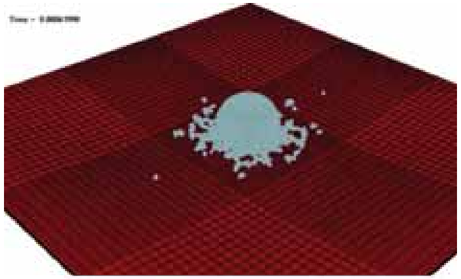
Experiment Image from Libersky et al (1993) J Comp Phys 109, 67-75



- Aircraft Ditching & extreme wave loading



- Fluid Sloshing
- Birdstrike & ice impact



- Fracture and fragmentation

Introduction

Properties of Smoothed Particle Hydrodynamics offer advantages for dealing with specific solid mechanics problems, for example:

- Impact loading
- Blast loading
- Manufacturing processes (forming, machining)

Advantages, owing to the method's meshless discretisation, are:

- Ability to deal with large deformations
- Reduced directional sensitivity
- Ability to deal with material failure and fracture

Introduction

Smoothed Particle Hydrodynamics method has other properties which can offer advantages in dealing with large deformation, damage and fracture problems:

- Non-local properties
  - These can be used to address the **localisation** and **material softening** problem, including a treatment of material softening due to damage, in order to reduce and ultimately remove mesh dependency;
  - Strain softening effects have been analysed in FEM and SPH method using CDM approach and alternative damage model;

- Particle-to-particle interactions
  - These can be used to model the formation of free surfaces and crack opening (fracture)

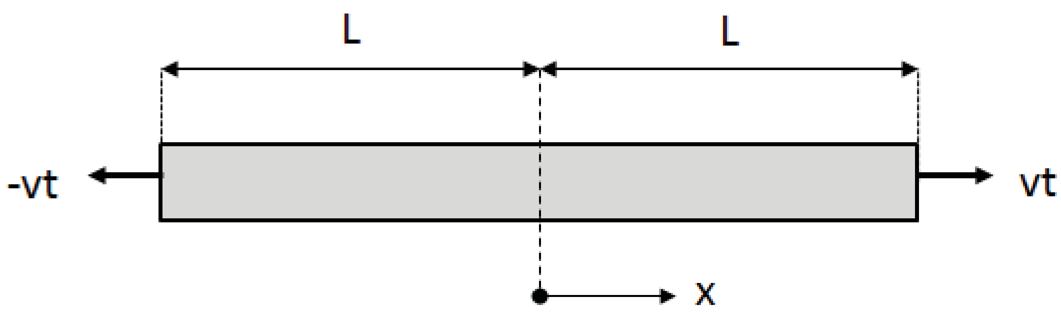
Non-local properties - Strain Softening Problem

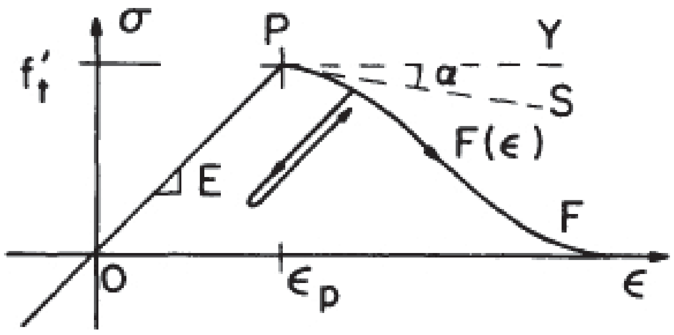
- In the framework of local continuum damage mechanics (CDM), damage is modelled as a degeneration of material properties induced by mechanical loading, which can result in strain softening behaviour;
- This leads to an ill-posed boundary value problem, where the local governing hyperbolic differential equations at a point become elliptic, which induces a numerical instability;
- This instability is mesh-sensitive and manifests itself as non-physical deformation of the softening continuum (due to deformation localisation, infinite number of bifurcated branches and post-bifurcation mesh dependency issues);
- This work primarily considered the strain-softening effects in the SPH spatial discretisation, and compares those with the classical CDM-FEM and equivalent damage force model solutions;

R. Vignjevic, N. Djordjevic, S. Gemkow, T. De Vuyst, J. Campbell, SPH as a nonlocal regularisation method: Solution for instabilities due to strain-softening, Comput. Methods Appl. Mech. Engrg. 277 (2014) 281–304, http://dx.doi.org/10.1016/j.cma.2014.04.010

Strain Softening Problem – FEM vs SPH

- One-dimensional dynamic strain softening problem, for which an exact analytical solution for wave propagation in the one-dimensional strain softening problem was known (Bažant and Belytschko, 1985)





$\dot{\varepsilon}_{ij}\dot{\sigma}_{ij} > 0$ Bifurcation criterion

$\dot{\sigma}_{ij} = \mathbb{C}_{ijkl}\dot{\varepsilon}_{kl}$ Rate form of constitutive equation

Material stability criterion

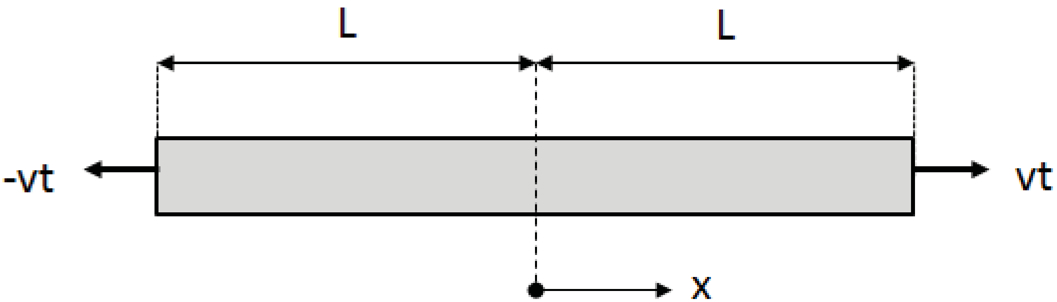
$$\boxed{\dot{\varepsilon}_{ij}\mathbb{C}_{ijkl}\dot{\varepsilon}_{kl} = 0}$$

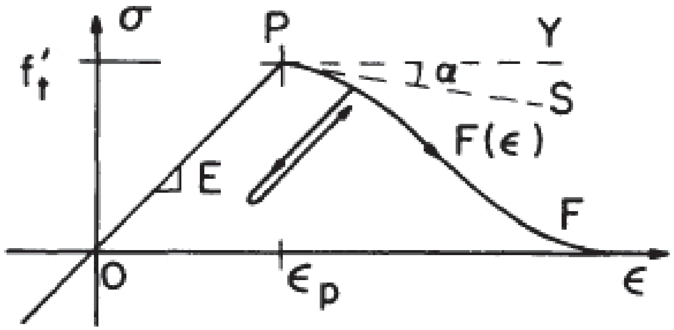
$$c^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$$ Elastic response

Softening response

$$c^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial t^2} = 0 \qquad c^2 = \frac{F'(\varepsilon)}{\rho}$$

Strain Softening Problem – FEM vs SPH





$$u(x,t) = -v\left\langle t - \frac{x+L}{c} \right\rangle + v\left\langle t + \frac{x-L}{c} \right\rangle$$

Analytical solution for displacement field

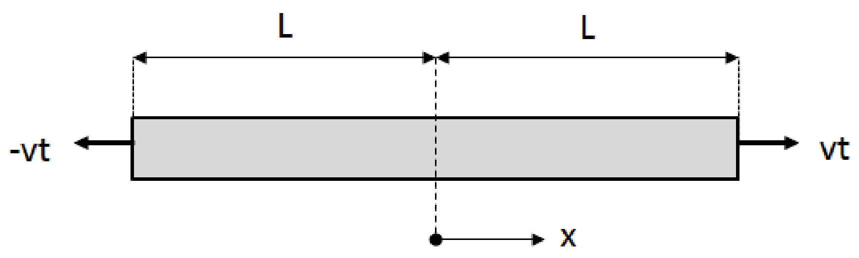
$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{v}{c}\left[H\left(t - \frac{x+L}{c} \right) + H\left(t + \frac{x-L}{c} \right) \right]$$

Strain field before the wave superposition in the midsection of the bar

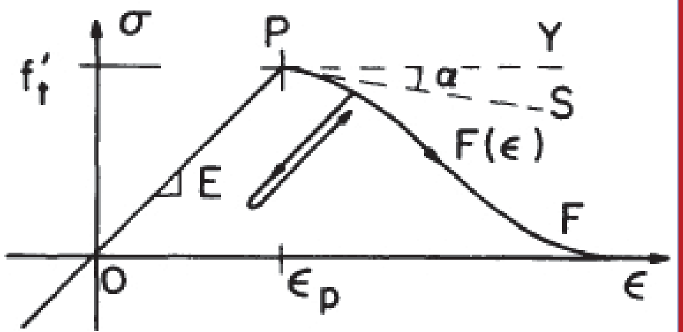
$$\varepsilon_x = \frac{v}{c}\left[H\left(t - \frac{x+L}{c} \right) - H\left(t + \frac{x-L}{c} \right) + 4v\langle t - L/c \rangle \delta(x) \right]$$

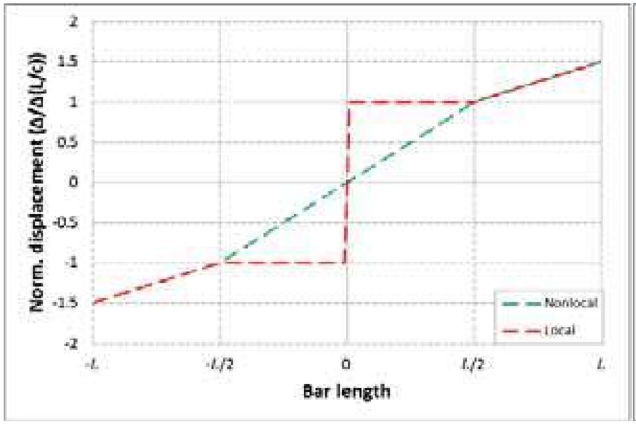
Strain field outside of the softening zone

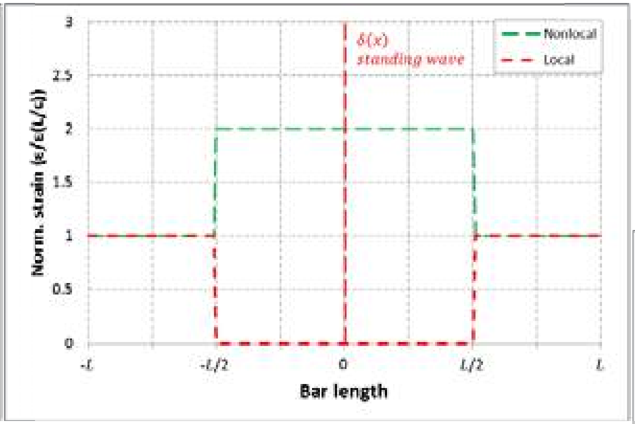
Strain Softening Problem – FEM vs SPH

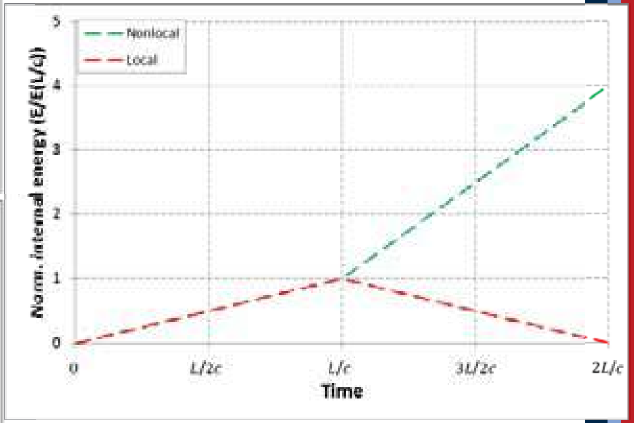


Geometry and loading of softening bar Bažant and Belytschko (1985)



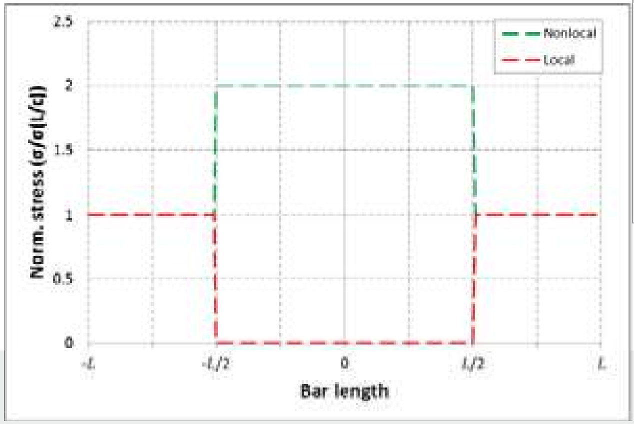






Elastic local and nonlocal solutions at the time instance

$$t = \frac{3}{2}\frac{L}{c}$$

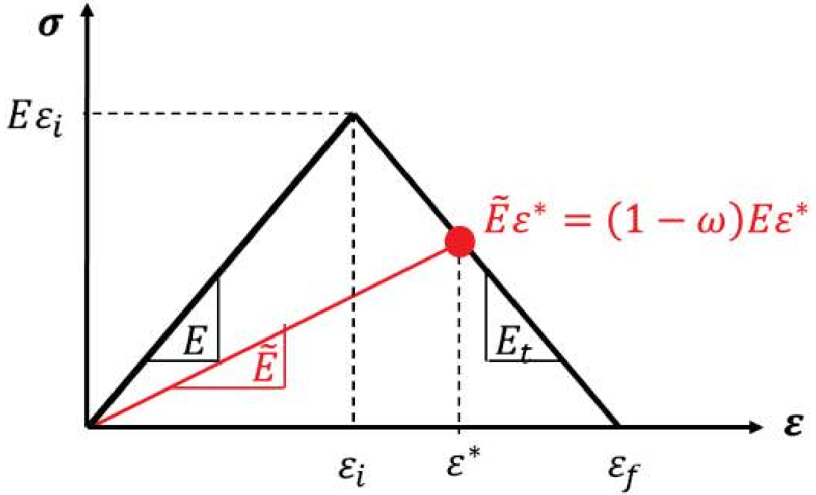


Strain Softening Problem – FEM vs SPH

- Bilinear constitutive law based on local continuum damage mechanics approach;

$$\bar{\sigma} = \frac{\sigma}{1-\omega} \qquad \omega = 1 - \frac{\varepsilon_i\left(\varepsilon_f - \varepsilon^*\right)}{\varepsilon^*\left(\varepsilon_f - \varepsilon_i\right)}$$

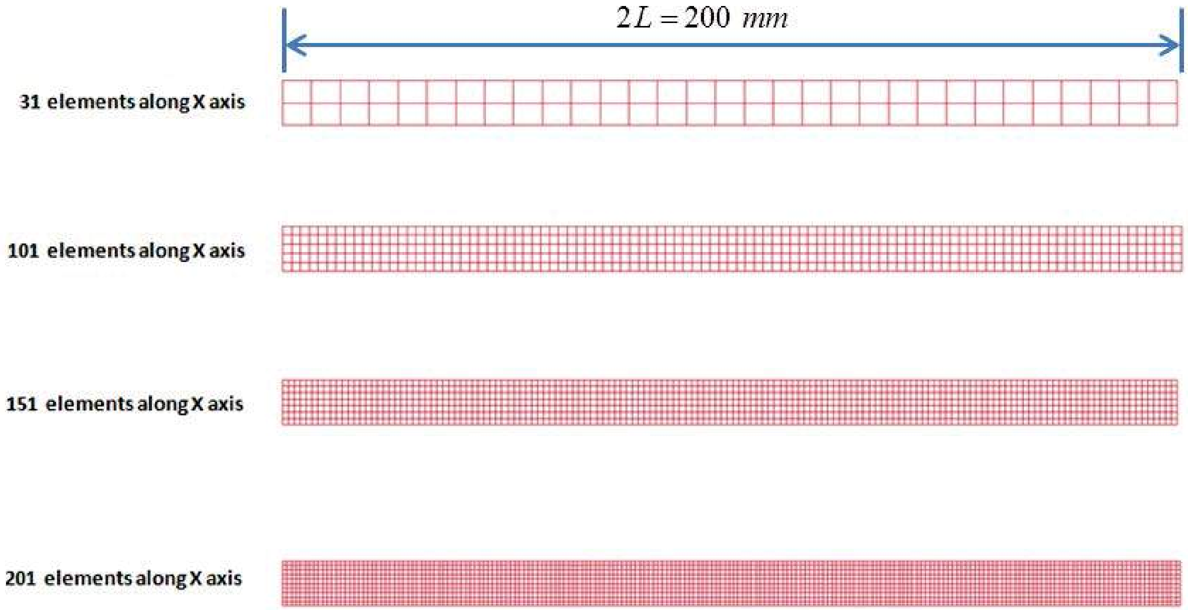
$$E_T = -\frac{E\varepsilon_i}{\varepsilon_f - \varepsilon_i}$$



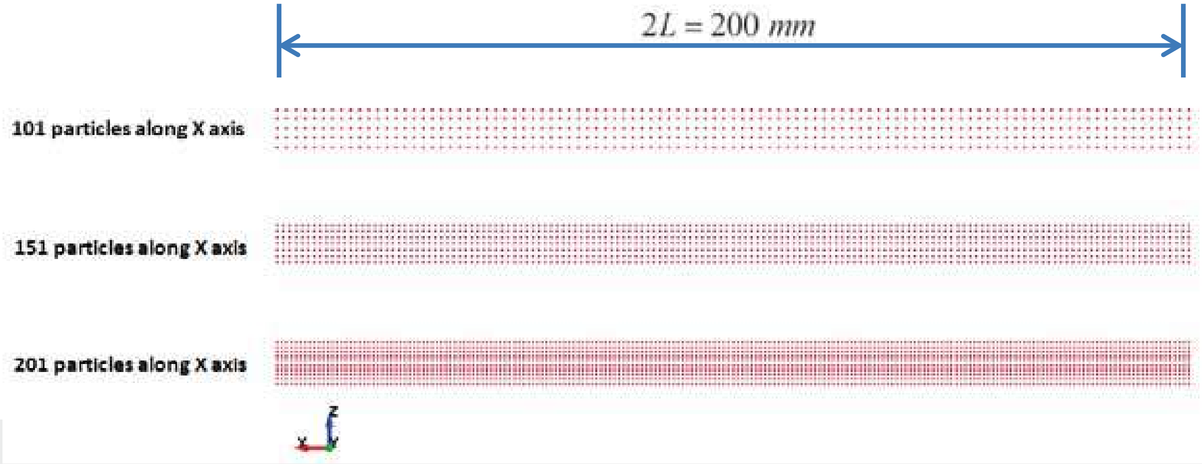
- Two discretisation methods:
  - Finite Elements Method (FEM);
  - Smooth Particle Hydrodynamics (SPH);

Strain Softening Problem – FEM vs SPH

Spatial discretisation used in the FEM (DYNA3D) simulation of the strain-softening bar

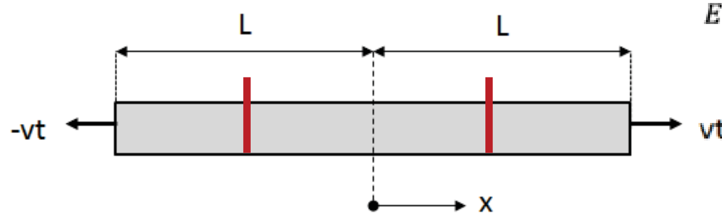


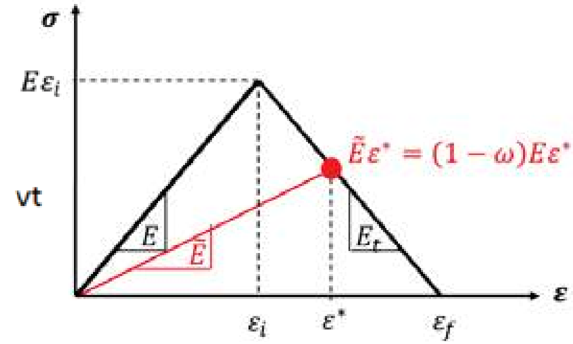
Particle discretisation in SPH (MCM) of strain-softening bar

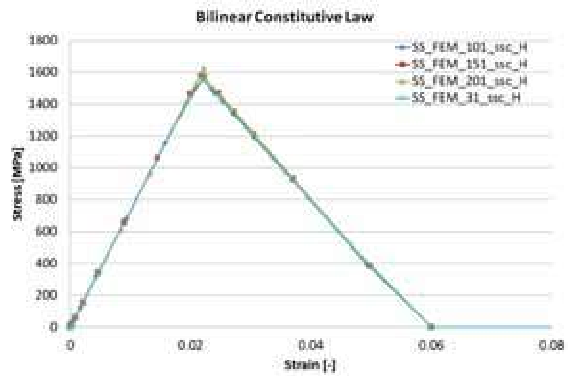


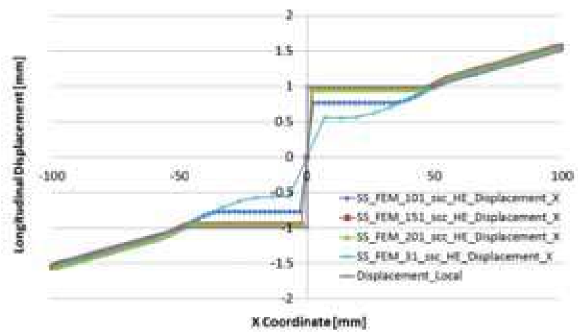
Strain Softening Problem – FEM Results

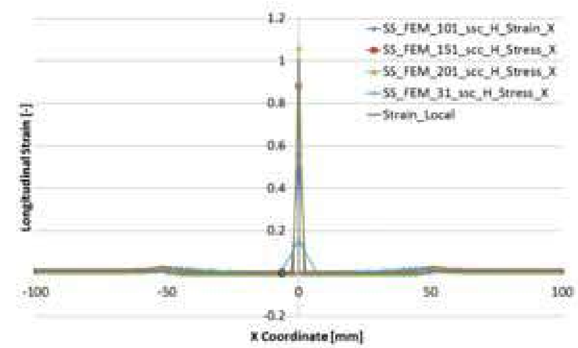
Bilinear law implemented in FEM using a damage parameter and classic CDM approach

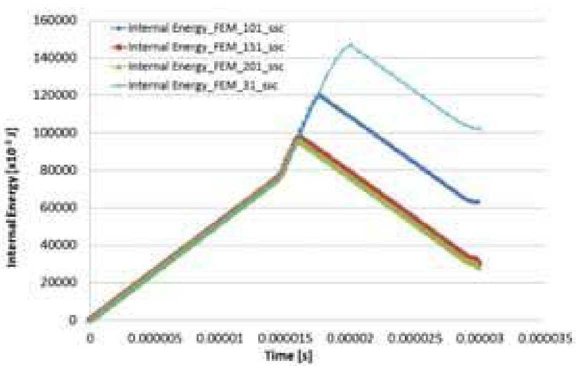


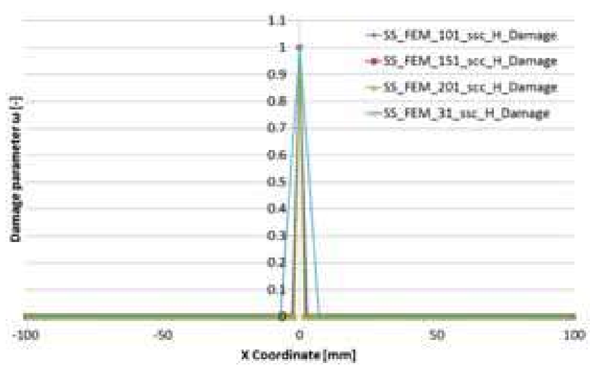


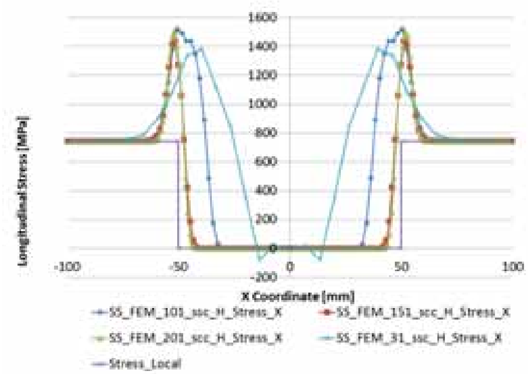






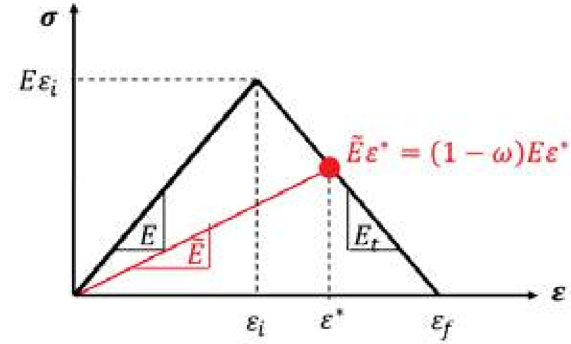


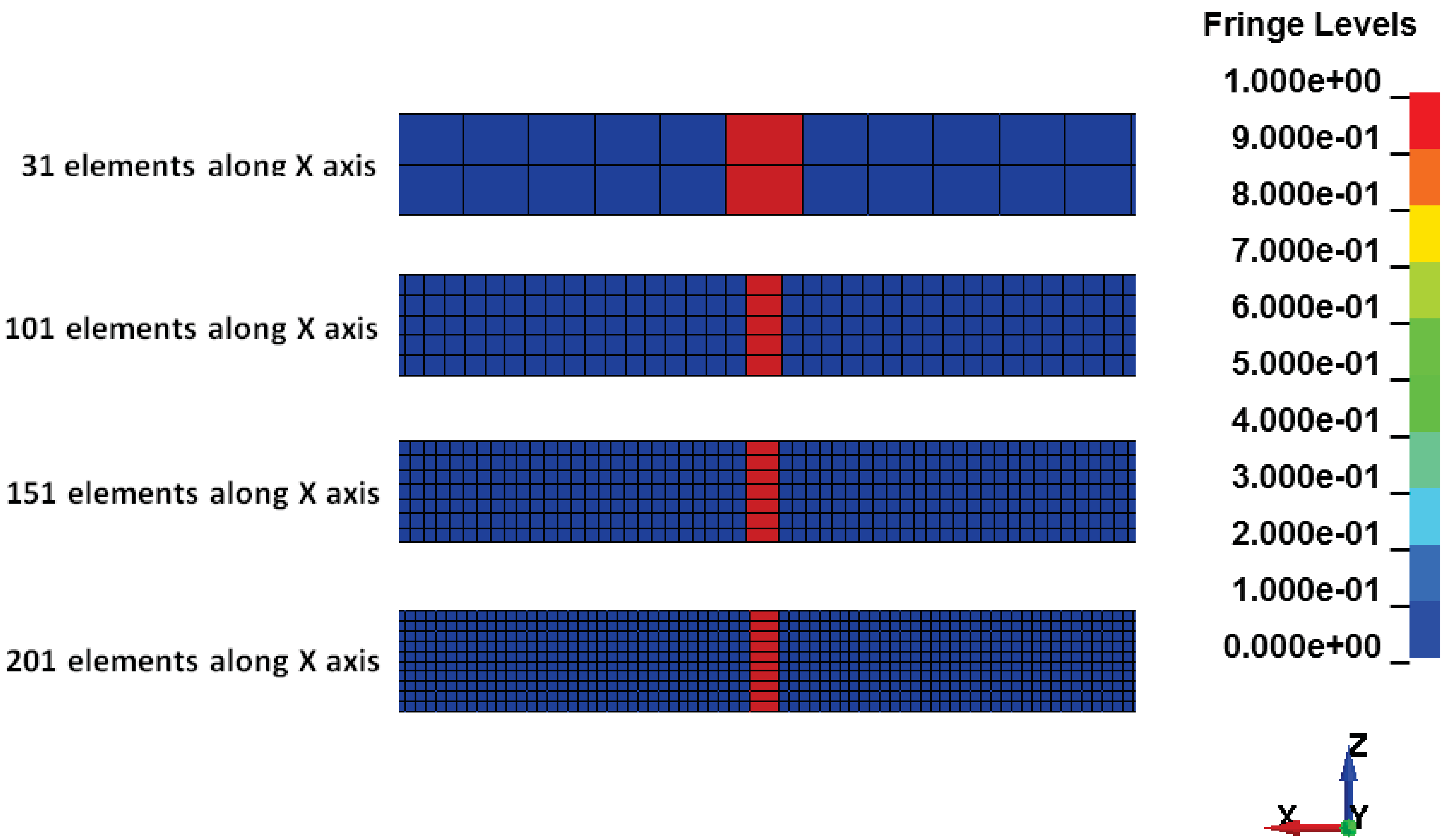




Damage localised in a single element

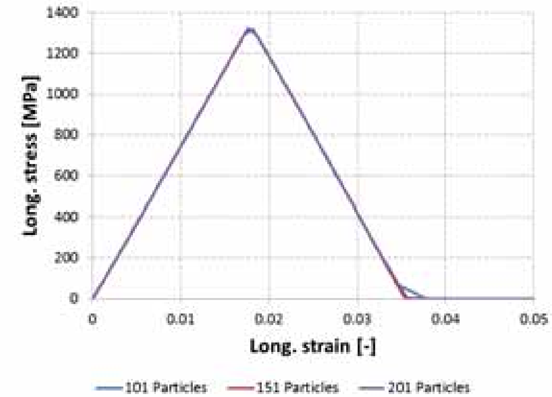
Strain Softening Problem – FEM Results





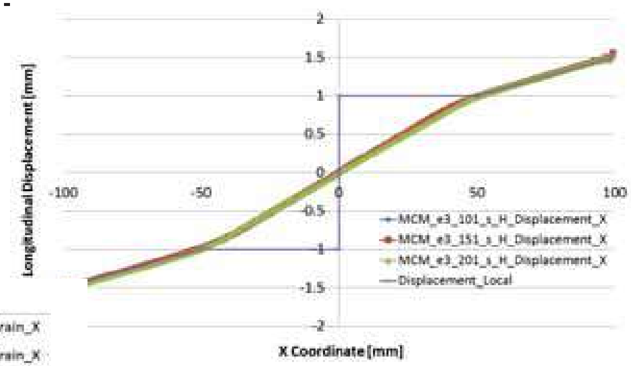
Damage localised in a single element

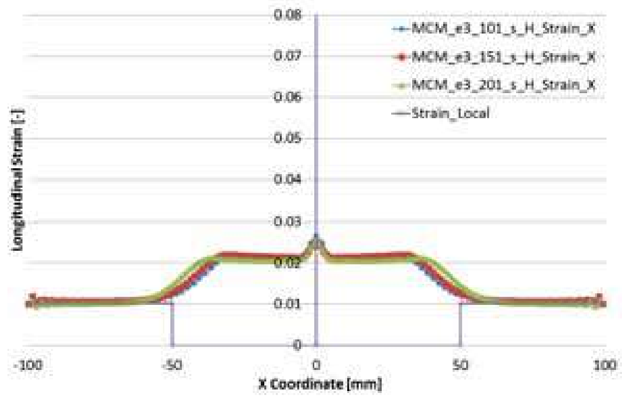
Strain Softening Problem – SPH Results (constant smoothing length)



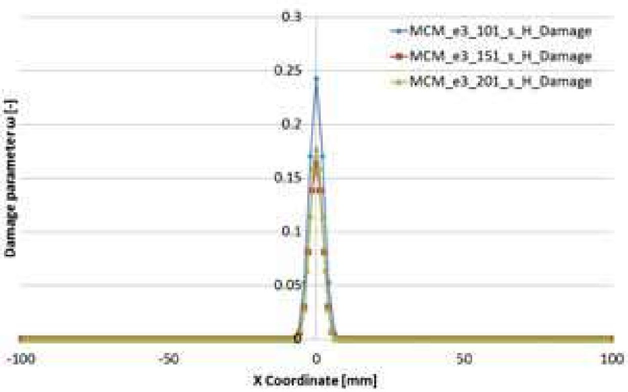
Longitudinal stress vs. longitudinal strain curves for the central particle for different values of Δp, SPH

Analytical solution and the SPH-experiment 3 numerical results for longitudinal displacement at $t = 3/2 \cdot L/c_e$

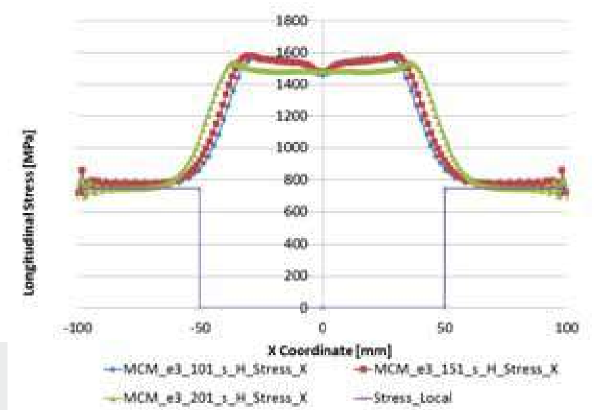




Analytical solution and the SPH-experiment 3 numerical results for longitudinal strain at $t = 3/2 \cdot L/c_e$



Damage distribution for different particle densities, SPH-experiment 3, response time $t = 3/2 \cdot L/c_e$



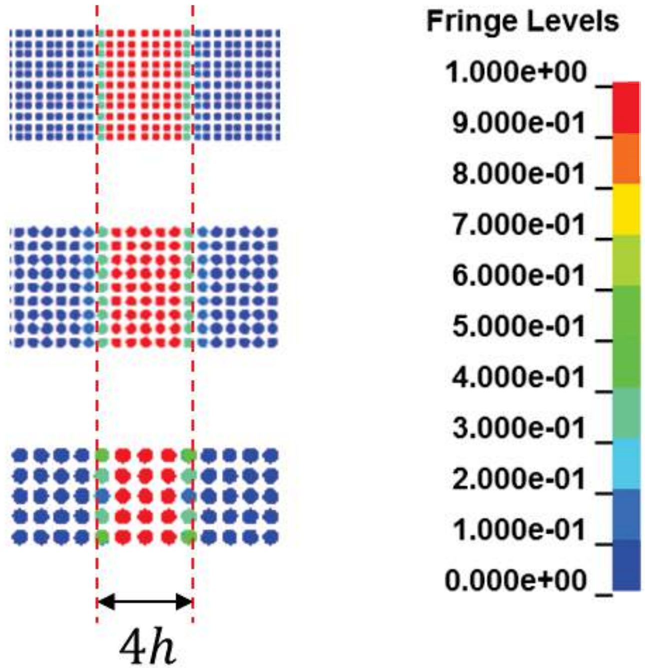
Analytical solution and the SPH-experiment 3 numerical results for longitudinal stress at $t = 3/2 \cdot L/c_e$

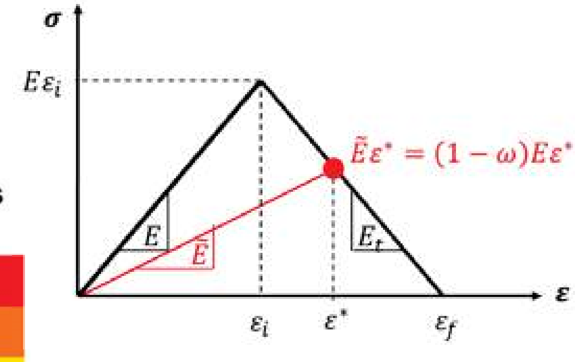
Strain Softening Problem – SPH Results (constant smoothing length)

$$\Delta p = \frac{200mm}{201}$$

$$\Delta p = \frac{200mm}{151}$$

$$\Delta p = \frac{200mm}{101}$$

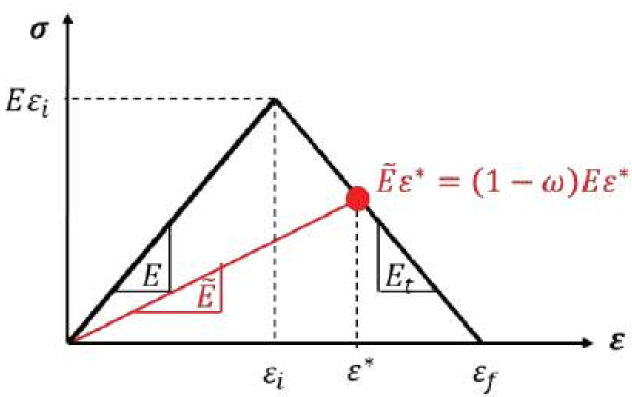




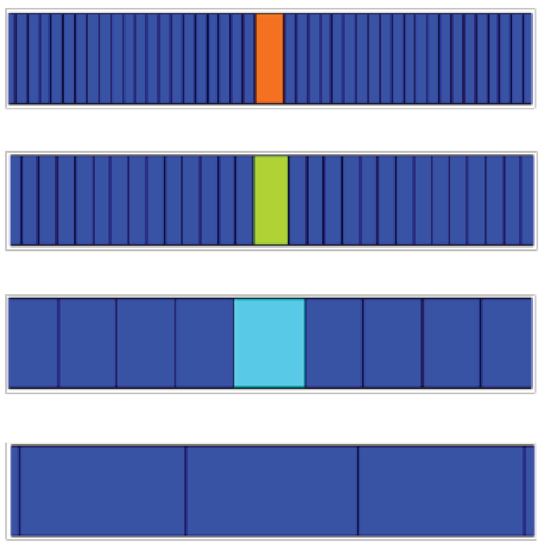
Damage distributed within a limited zone with 4h size around the bar symmetry plane at response time $t = \frac{3}{2}\frac{L}{c}$

Strain Softening Problem – SPH Results (constant smoothing length)

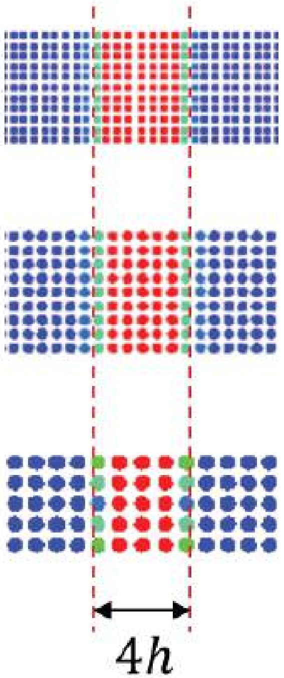
SPH as Non-local Regularisation Method



FEM



SPH



R. Vignjevic, N. Djordjevic, S. Gemkow, T. De Vuyst, J. Campbell, SPH as a nonlocal regularisation method: Solution for instabilities due to strain-softening, Comput. Methods Appl. Mech. Engrg. 277 (2014) 281–304, http://dx.doi.org/10.1016/j.cma.2014.04.010

Non-local properties

In continuum damage mechanics, material damage can result in strain-softening behaviour.

- Leads to ill-posed boundary value problem in standard FEM
- Solved by using non-local integral regularisation methods
- SPH is inherently non-local

Non-local properties can also be used to reformulate CDM problems by introducing an equivalent damage force vector.

R. Vignjevic, N. Djordjevic, S. Gemkow, T. De Vuyst, J. Campbell, SPH as a nonlocal regularisation method: Solution for instabilities due to strain-softening, Comput. Methods Appl. Mech. Engrg. 277 (2014) 281–304, http://dx.doi.org/10.1016/j.cma.2014.04.010

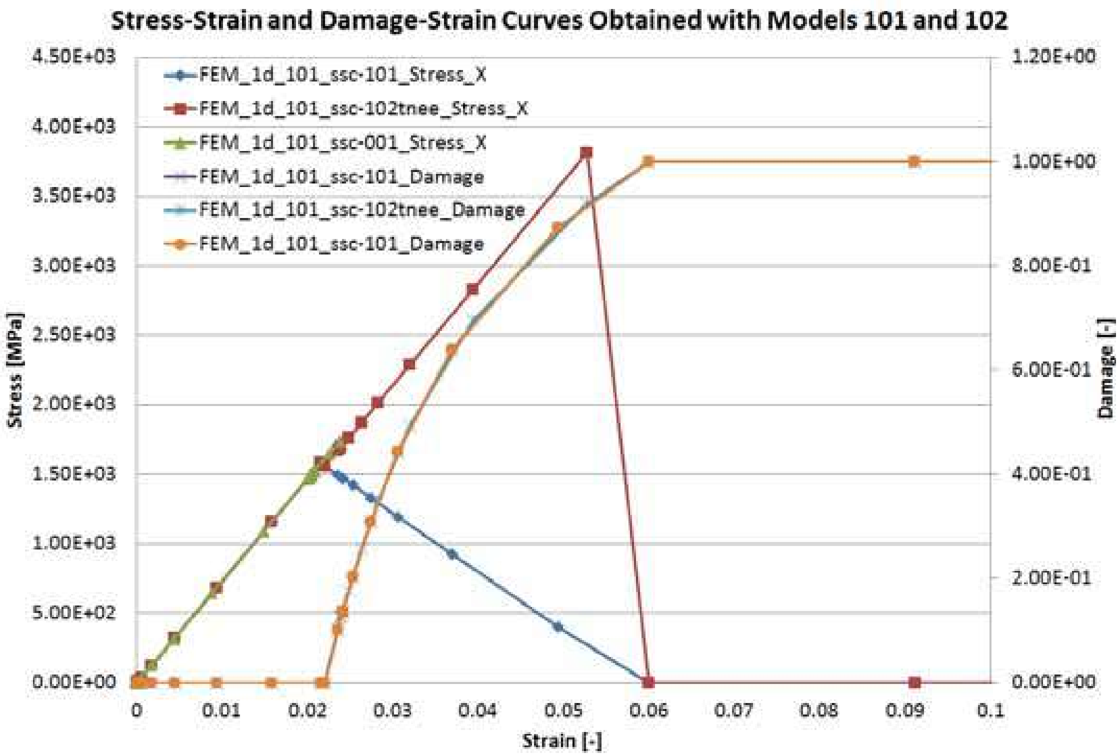
R. Vignjevic, N. Djordjevic, T.D. Vuyst, S. Gemkow, Modelling of strain softening materials based on equivalent damage force, *Comput. Methods Appl. Mech. Engrg.* (2018), https://doi.org/10.1016/j.cma.2018.01.049

Strain Softening Problem - FEM with Equivalent Damage Force

$$\bar{\sigma} = \frac{\sigma}{1-\omega} \qquad \nabla\sigma = \nabla\bar{\sigma} - \nabla\left(\omega\bar{\sigma}\right)$$

Conservation of momentum

$$\nabla\sigma + b = \rho a$$



Damage parameter in bilinear constitutive law

$$\omega = \frac{\varepsilon_f\left(\varepsilon^* - \varepsilon_i\right)}{\varepsilon^*\left(\varepsilon_f - \varepsilon_i\right)}$$

Strain Softening Problem - FEM with Equivalent Damage Force

$$\nabla\sigma + b = \rho a$$

Conservation of momentum

$$\nabla\sigma = \nabla\bar{\sigma} - \nabla\left(\omega\bar{\sigma}\right)$$

$$\omega = \frac{\varepsilon_f\left(\varepsilon^* - \varepsilon_i\right)}{\varepsilon^*\left(\varepsilon_f - \varepsilon_i\right)}$$

FEM formulation weak form:

$$\int_\Omega \rho\{\delta w\}^T\{\ddot{u}\}dV + \int_\Omega \{\nabla\cdot\delta w\}^T\{\bar{\sigma}\}dV - \int_\Omega \{\delta w\}^T\{b\}dV - \int_\Gamma \left(\{\delta w\}^T\{\bar{\sigma}\}\right)\cdot n d\Gamma +$$

$$\boxed{\int_\Omega \{\delta w\}^T\{\omega\}\{\nabla\cdot\bar{\sigma}\}dV + \int_\Omega \{\delta w\}^T\{\nabla\cdot\omega\}\{\bar{\sigma}\}dV = 0}$$

Equivalent damage force:

$$\{f\}_D = \int_\Omega [N]^T\{\nabla\cdot\omega\}\{\bar{\sigma}\}dV + \int_\Omega [N]^T\{\omega\}\{\nabla\cdot\bar{\sigma}\}dV$$

Strain Softening Problem - FEM with Equivalent Damage Force

Equivalent damage force

$$\{f\}_D = \int_\Omega [N]^T \boxed{\{\nabla \cdot \omega\}} \{\bar{\sigma}\} dV + \int_\Omega [N]^T \{\omega\} \boxed{\{\nabla \cdot \bar{\sigma}\}} dV$$

Requires calculation of

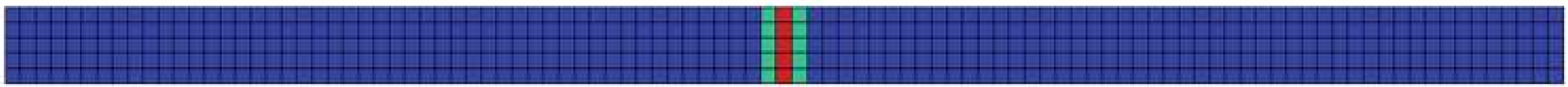
$$\nabla \cdot \omega$$

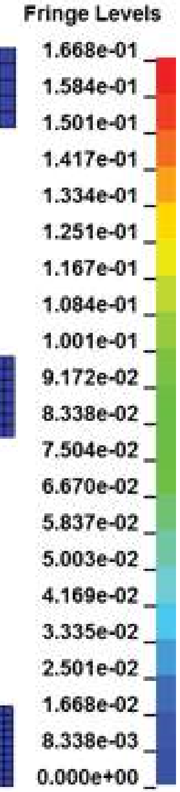
$$\nabla \cdot \bar{\sigma}$$

This can be done using SPH interpolation over neighbouring Gauss quadrature points:

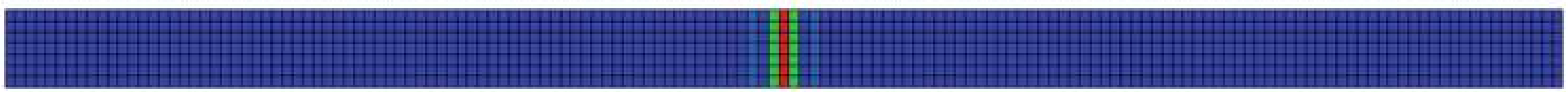
$$\nabla f(\boldsymbol{x}_I) = \sum_J \frac{m_J}{\rho_J} f(\boldsymbol{x}_J) \left[\nabla W\left(|\boldsymbol{x}_I - \boldsymbol{x}_J|, l_\omega \right) \right]$$

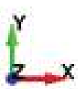
Strain Softening Problem - FEM with Equivalent Damage Force



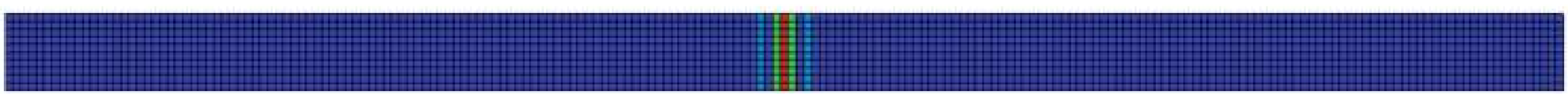


101 linear elements along loading direction





151 linear elements along loading direction



201 linear elements along loading direction

Strain Softening Problem - Summary

- The strain softening process in the classic CDM combined with FEM is localised in a single element and cannot propagate;
- Localisation effects related to material strain-softening are not present with the SPH method – the method inherently nonlocal;
- Size of the softening zone was defined by the smoothing length and for a fixed smoothing length h, the stain softening was independent of the particle density;
- Modelling of damage by using Equivalent Damage Force addresses the localisation problem, with the size of damaged zone determined by input parameter called damage characteristic length;

R. Vignjevic, N. Djordjevic, T.D. Vuyst, S. Gemkow, Modelling of strain softening materials based on equivalent damage force, *Comput. Methods Appl. Mech. Engrg.* (2018), https://doi.org/10.1016/j.cma.2018.01.049

Particle interaction area

- An alternative approach to modelling damage in SPH, based on the weakening the interparticle interactions combined with the visibility criterion
- The concept of area vectors within the SPH method was outlined by Swegle* for the purpose of discussing the tensile instability inherent in any basic SPH description;
- Swegle noted that the fundamental definition of the stress tensor shows that a force exerted on a surface due to stress is given by:

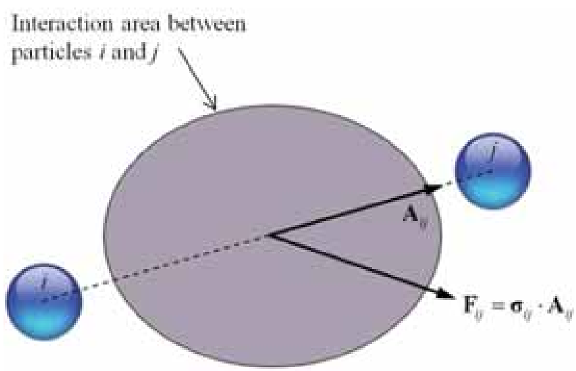
$$\mathbf{F} = \boldsymbol{\sigma} \cdot \mathbf{A}$$

- The conservation of momentum equation can be rewritten as

$$F_i = m_i a_i = -\sum_j m_i m_j \left[\frac{\boldsymbol{\sigma}_i}{\rho_i^2} + \frac{\boldsymbol{\sigma}_j}{\rho_j^2} \right] \nabla_i W_{ij}$$

- Rearranging allows the definition of an interaction area:

$$F_i = -\sum_j \left[\left(\boldsymbol{\sigma}_i\right) \frac{\rho_j}{\rho_i} + \left(\boldsymbol{\sigma}_j\right) \frac{\rho_i}{\rho_j} \right] A_{ij}$$



$$A_{ij} = V_i V_j \nabla_i W_{ij}$$

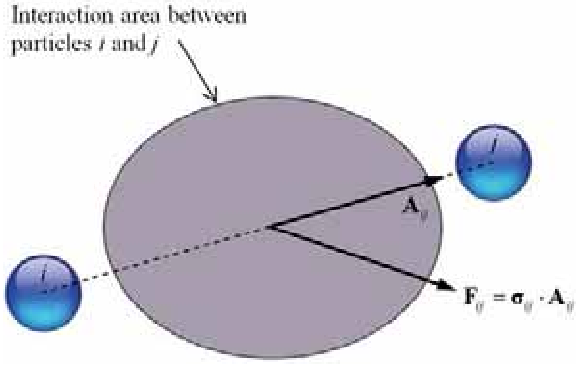
* J W Swegle. SAND2000-1223, Sandia National Laboratories (2000)

Particle interaction area

- Damage ω_{ij} is defined as an interparticle parameter that reduces the interaction area A_{ij}
- Modification of the stress tensor is not required:

$$F_i = -\sum_j \left[(\boldsymbol{\sigma}_i)\frac{\rho_j}{\rho_i} + (\boldsymbol{\sigma}_j)\frac{\rho_i}{\rho_j} \right] A_{ij} \left(1 - \omega_{ij}\right)$$

- Current implementation is based simple damage and failure models;

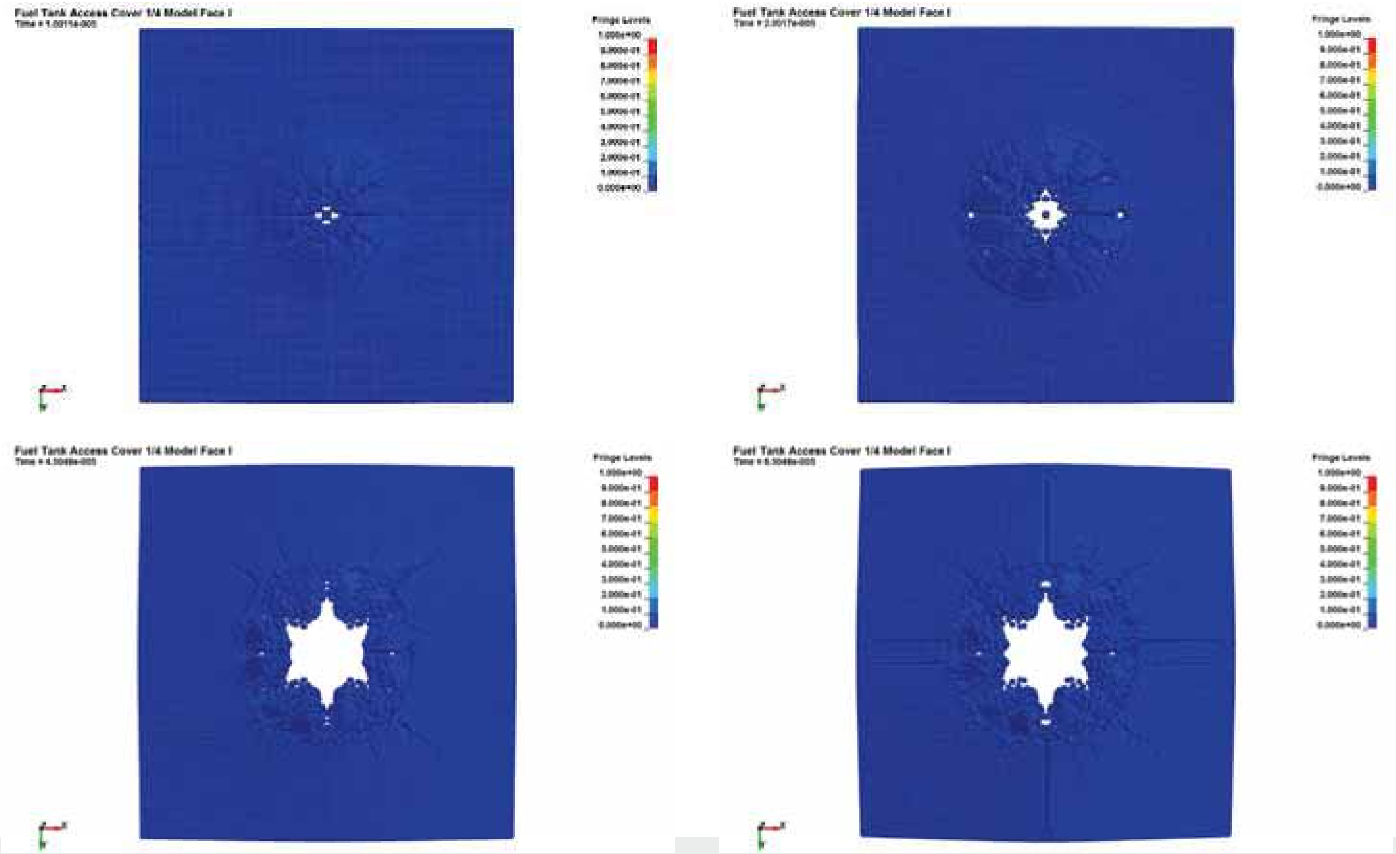


Particle interaction area

- Tensile test of ductile material (plasticity) – dog bone specimen;
- Failure model with mapping of internal state variable (effective plastic strain);

Particle interaction area

- Impact on brittle material (isotropic elastic with failure stress)
- Quarter model of 6.6g cube impacting brittle plate – evolution of fractures



Particle interaction area

- Impact on brittle material modelled with isotropic elastic with failure stress;
- Quarter model of 6.6g cube impacting brittle plate;

Particle interaction area summary

- A damage modelling approach consistent with classical continuum damage mechanics but which does not require the use of an effective stress to include damage has been developed in SPH:
  - The SPH momentum equation rearranged in a way that it contained a particle-particle interaction area, which the damage is applied to;
  - Inter-particle damage approach works with ductile and brittle failure for a range of loading conditions;

Summary and Outlook

Summary

- SPH has non-local properties allowing solution of strain softening problems
- SPH can be used to calculate an equivalent damage force in FEM
- SPH particle interaction area can be used to model generation and propagation of cracks

Outlook

- Extend developed techniques to damage models for ductile metals and fibre reinforced composite materials

Smoothed Particle Hydrodynamics method for Plant tissue growth mechanics

2018 SPH SIG Meeting

Matthias MIMAULT

The James Hutton Institute

12 April 2018

Introduction

Plant root system

Roots are an essential component of the plant system

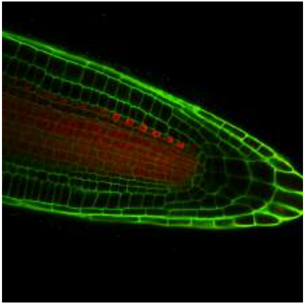
- Invisible development
- Dense network
- Complex architecture
- Autonomous design
- Resilient anchorage

Introduction

Focus: Growth mechanisms of meristem

Important tissue, located at the tip of the roots

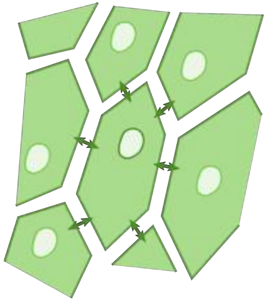
- Cell development
- Bacterial colonisation
- Ground excavation
- Chemical transport hub

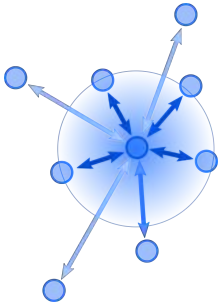


Introduction

Smoothed Particle Hydrodynamics and biology

<u>Proposition:</u> Identify the cells to the particle kernels



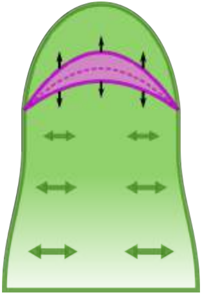


- Reproduction of neighbour interactions
- Inclusion of micro-mechanisms in macro-stress

Meristem description

Three growth mechanisms are identified

- Cell division
- Cell extension
- Mass assimilation

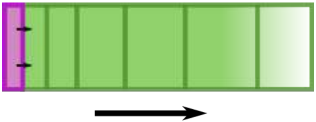


Model

Meristem description

Modelling choices for living material

- Solid dynamics
- Weakly comp. SPH
- Particle source
- Poroelasticity
- Global density source



Model

Balance equations

Equation for density and momentum conservation

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = -\rho\nabla\cdot\boldsymbol{u} + \gamma$$
$$\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = \frac{\nabla\cdot\sigma}{\rho}$$

- Density ρ
- Source mass γ
- Velocity vector $\boldsymbol{u}$
- Stress σ

Model

Constitutive equations

Expression of pressure and source terms

$$\sigma = -(P + p)\mathbb{I} + \tau$$

$$\gamma = \lim_{\Delta t \to 0} \frac{\rho_0 - \rho(t)}{\Delta t}$$

- Hydrostat. pressure P
- Pore pressure p
- Shear stress τ
- Reference density ρ_0

Model

Constitutive equations

The stress is decomposed in

$$P(\rho) = c_0^2 \left(\rho_0 - \rho\right)$$
$$\frac{\mathrm{d}\hat{\tau}_{ij}}{\mathrm{d}t} = 2\mu \left(\dot{\varepsilon}_{ij} - \frac{1}{3}\delta_{ij}\varepsilon_{kk}\right) + \omega_{ik}\tau_{kj} - \tau_{ik}\omega_{kj}$$
$$p(x) = \alpha_p x e^{-\frac{x^2}{\beta_p}}$$

- Weakly-compressible framework for P
- Jaumann derivative for τ
- Bell shape distribution with a tail for p

SPH implementation

Discrete equations

SPH reformulation of the equations

$$\langle -\rho \nabla \cdot u \rangle_a = \sum_b m_b^n \left(u_a - u_b \right) \cdot \nabla_a W_{ab},$$

$$\left\langle \frac{\nabla \cdot \sigma}{\rho} \right\rangle_a = \sum_b m_b^n \left(\frac{\sigma_a}{\rho_a^2} + \frac{\sigma_b}{\rho_b^2} + \Pi_{ab} \mathbb{I} \right) \cdot \nabla_a W_{ab}$$

- Wendland kernel W_{ab}
- Artificial viscosity Π_{ab}
- Variable mass m_a^n

SPH implementation

Time integration

Implementation with Euler explicit scheme

$$m_a^{n+1} = m_a^n + \Delta t\, m_a^n \left(\frac{\rho_0}{\rho_a^n} - 1\right)$$

$$\tau_a^{n+1} = \tau_a^n + \Delta t \left\langle \frac{\hat{\mathrm{D}\tau}}{\mathrm{D}t} \right\rangle_a$$

$$\rho_a^{n+1} = \rho_a^n + \Delta t \left(\left\langle \frac{\mathrm{D}\rho}{\mathrm{D}t} \right\rangle_a + \langle \gamma \rangle_a \right)$$

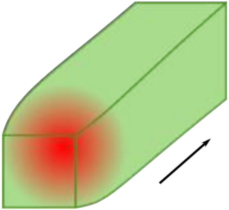
$$u_a^{n+1} = u_a^n + \Delta t \left\langle \frac{\nabla \cdot \sigma}{\rho} \right\rangle_a$$

Tests

Settings

Square-shape root with particle source and fixed pore pressure

- Timed source of particles
- Spatial distrib. pore pressure
- Steady-state
- DualSPHysics 4.0 CPU



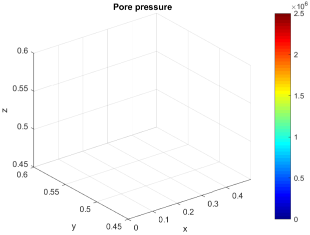
T	5.0
λ_{CFL}	0.2
α_a	0.5

Tests

Illustrative animation

Plot of pore pressure distribution

- Lateral expansion
- Steady-state
- Stable envelop
- Strong inner instabilities

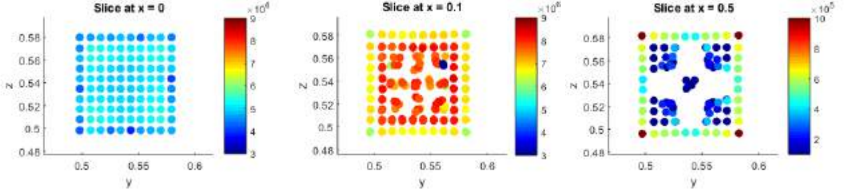


Evolution of pore pressure

Tests

Stress distribution

Samples of the Von Mises stress distribution

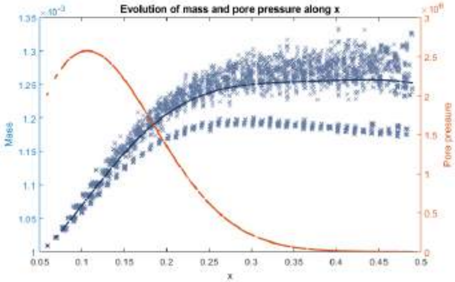


- At max pore pressure, strong stress from inside
- After release, stress is naught & extension kept

Tests

Mass evolution

Comparison of pore pressure and mass distributions



- 7-order polynomial regression for mass
- Correlation pore pressure-mass variation

Tests

Conclusion

3D model of tissue mechanics with cellular growth

- Solid SPH formulation
- Poroelasticity dynamics
- Mass agglomeration

Future works

- Stability: artificial tension, variable smoothing length
- Biology: multiphysics, cell division
- Code: GPU extension, real data integration

Thank you for your attention !

Contact: matthias.mimault@hutton.ac.uk

SPH Sampling and its applications in Computer Graphics

Richard Southern (with **Min Jiang** and **Declan Russell**)
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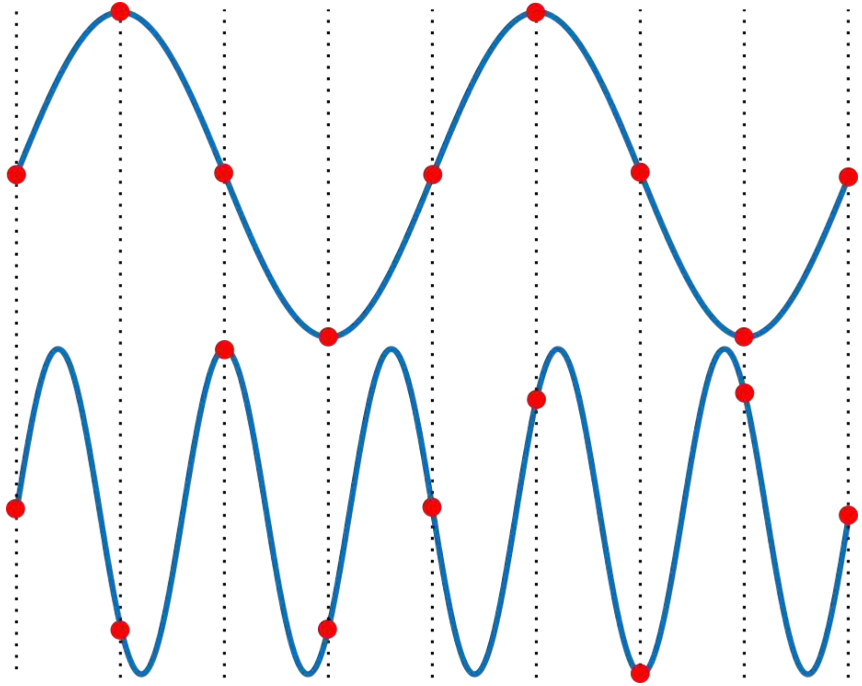
On Sampling



Sampled at Nyquist limit

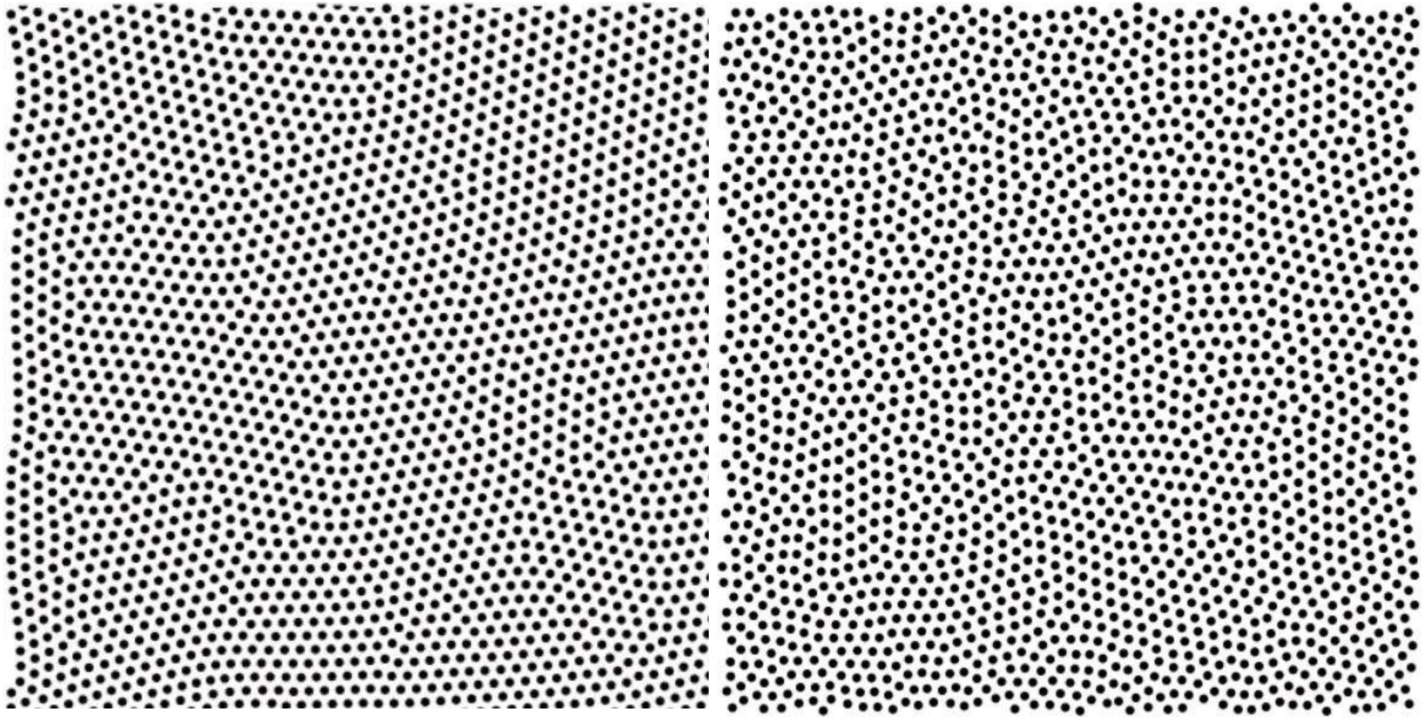
Sampled below Nyquist limit

Importance of Sampling in CG

- **Apophenia**: tendency to see patterns: distracting in **digital half-toning**[1]
- In **rendering**: strobing, jaggies, Moire patterns[2]
- In **simulation**: continuity of boundary interaction[3]

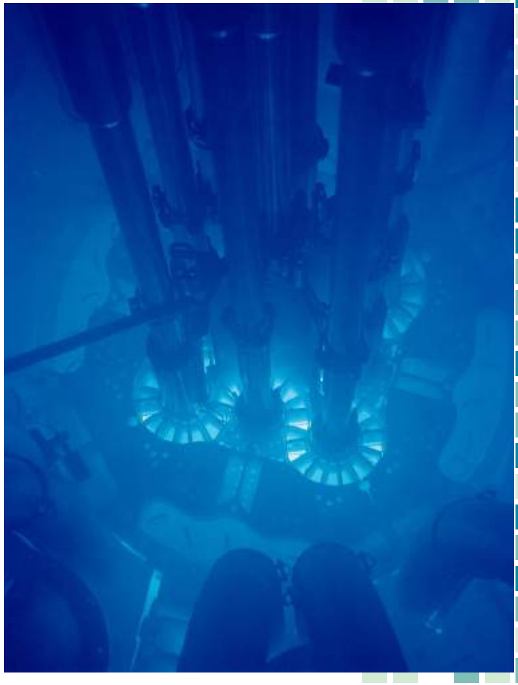
1. Ulichney R., **A Review of Halftoning Techniques**, http://www.hpl.hp.com/research/isl/halftoning/publications/2000-halftoning-review.pdf
2. Cook R., **Stochastic sampling in computer graphics**, https://dl.acm.org/citation.cfm?id=8927
3. Colagrossi A. *et al*, **Particle packing algorithm for SPH scheme**, https://www.sciencedirect.com/science/article/pii/S0010465512001051

Apophenia in action

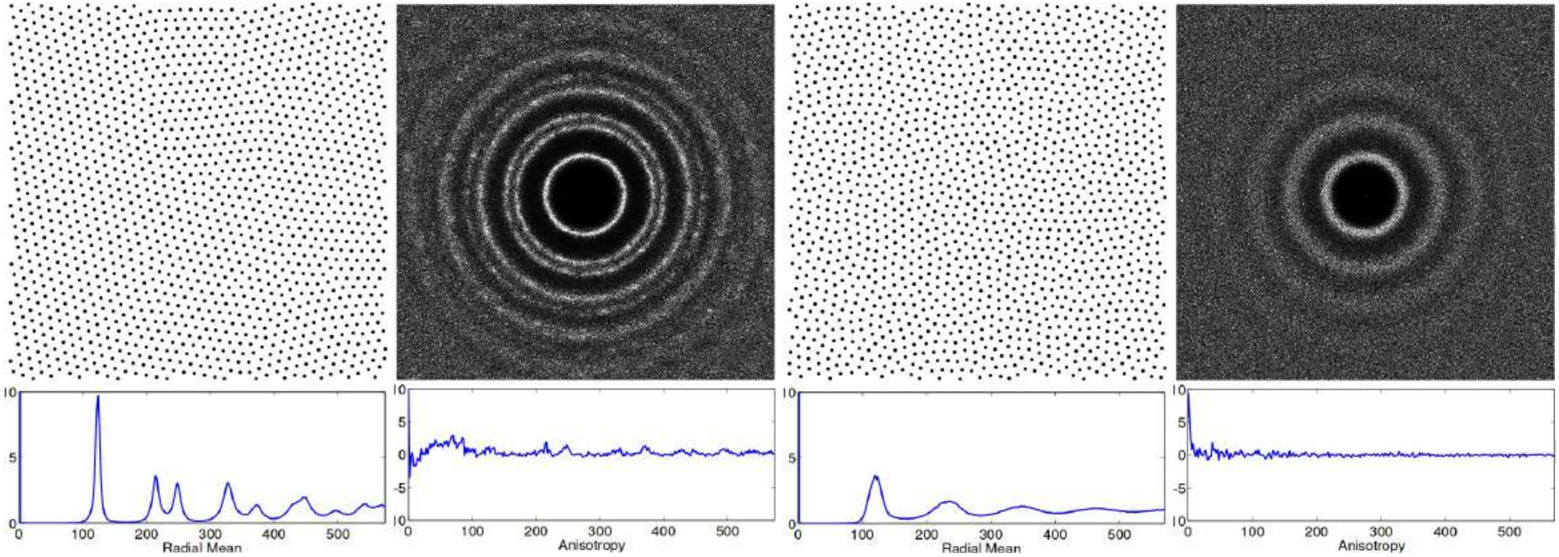


Blue Noise Sampling

- Colour used to describe types of noise
- **Blue Noise**: inverse frequency power spectrum distribution
- Minimal low frequency components and no spikes of energy: **even distribution, no clumps**
- Naturally occurring, e.g. retinal cells, radiation distribution, music



Flavours of Blue Noise



Effective Nyquist Frequency: size of “flat spot” **Oscillation:** magnitude of ripples[4]

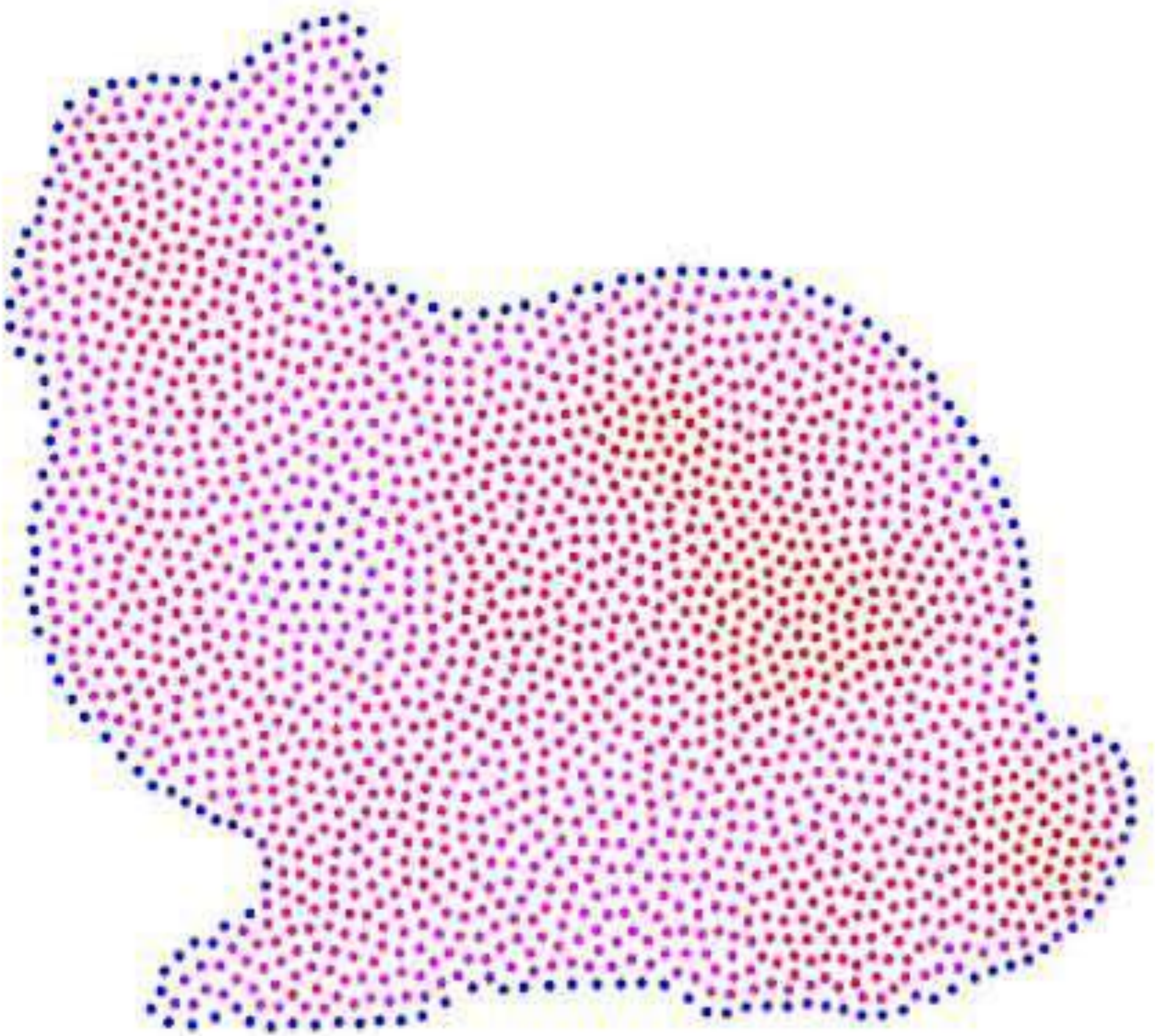
4. Heck D. *et al.*, **Blue noise sampling with controlled aliasing**, https://dl.acm.org/citation.cfm?id=2487233

Generating Blue Noise Profiles

- Poisson disk (dart throwing)[5]
- Voronoi tessellation[6]
- Tiling[7]
- Physically-based methods[8]

5. Bridson R., **Fast Poisson Disk Sampling in Arbitrary Dimensions,** https://dl.acm.org/citation.cfm?id=1278807
6. Balzer M., **Capacity-constrained point distributions: a variant of Lloyd's method**, https://dl.acm.org/citation.cfm?id=1531392
7. Kopf J., **Recursive Wang tiles for real-time blue noise**, https://dl.acm.org/citation.cfm?id=1141916
8. Wong K. M. et al., **Blue noise sampling using an N-body simulation-based method**, https://link.springer.com/article/10.1007/s00371-017-1382-9

Smoothed Particle Hydrodynamic Sampling



SPH Sampling

- SPH simulations are producing particle distributions to equalise density / pressure
- **Are they useful?**
- Investigated and quantified in [9]
- Start by considering *convergence criteria* for SPH

9. Jiang M. *et al.*, **Blue Noise Sampling using an SPH-based method**, http://dx.doi.org/10.1145/2816795.2818102

Pressure force in SPH

In standard WCSPH formulation, **pressure gradient** is:

$$\mathbf{F}_i^{\text{pressure}} = -\sum_j m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} \right) \nabla W_{ij}$$

with pressure defined by Tait equations of state[10]:

$$p = B\left[(\rho/\rho_0)^\gamma - 1\right]$$

or for the ideal gas (with **γ** = 1) simply:

$$p = k(\rho - \rho_0)$$

10. Monaghan J.J., **Simulating Free Surface Flows with SPH**, https://www.sciencedirect.com/science/article/pii/S0021999184710345

SPH Convergence

As SPH converges to steady state, 3 things happen:

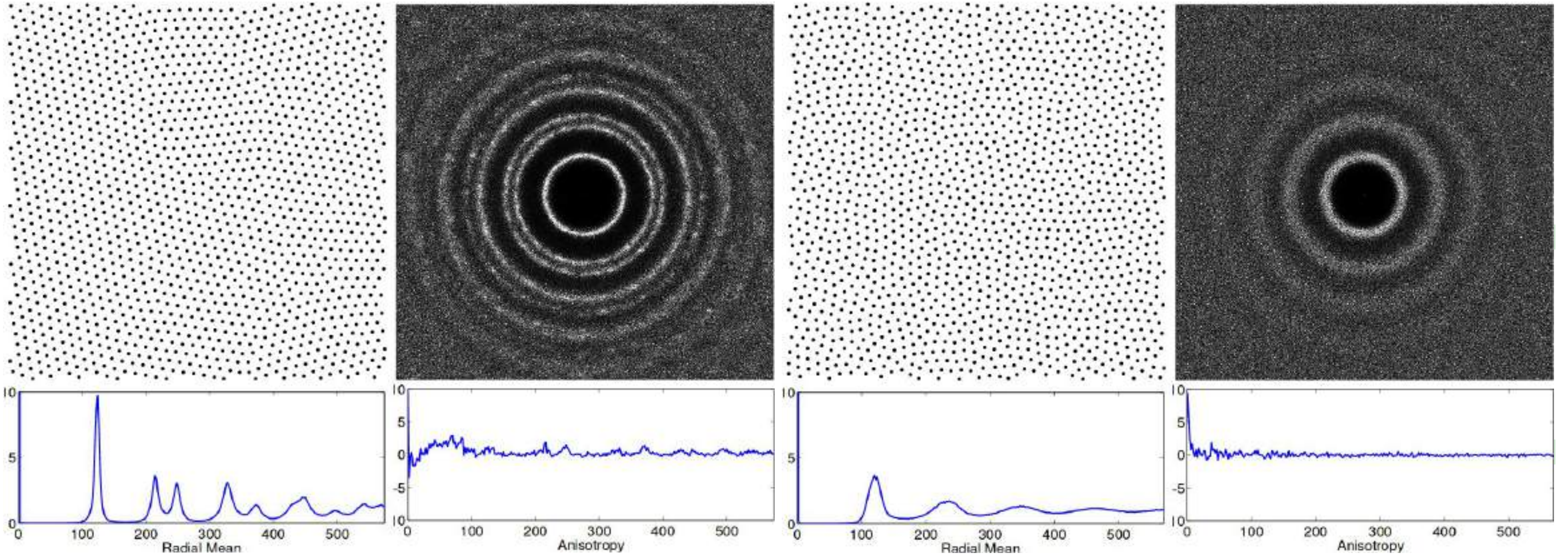
1. $\|x_{i+1} - x_i\| < \varepsilon,\ \varepsilon\,\text{small}$
2. $\forall i, j\ \rho_i \approx \rho_j$
3. $\forall i\ \mathbf{F}_i^{\text{pressure}} = 0$

Assuming mass is the same, we can write:

$$\mathbf{F}_i^{\text{pressure}} = 0 \approx \frac{2mk(\rho_i - \rho_0)}{\rho_i^2} \sum_j \nabla W_{ij}$$

$$\Rightarrow \sum_j \nabla W_{ij} = 0 \ \text{ or } \ \rho_i = \rho_0$$

Flavours of Blue Noise



$$\sum_j \nabla W_{ij} \Rightarrow 0$$

$$\rho_i \Rightarrow \rho_0$$

Manipulating convergence

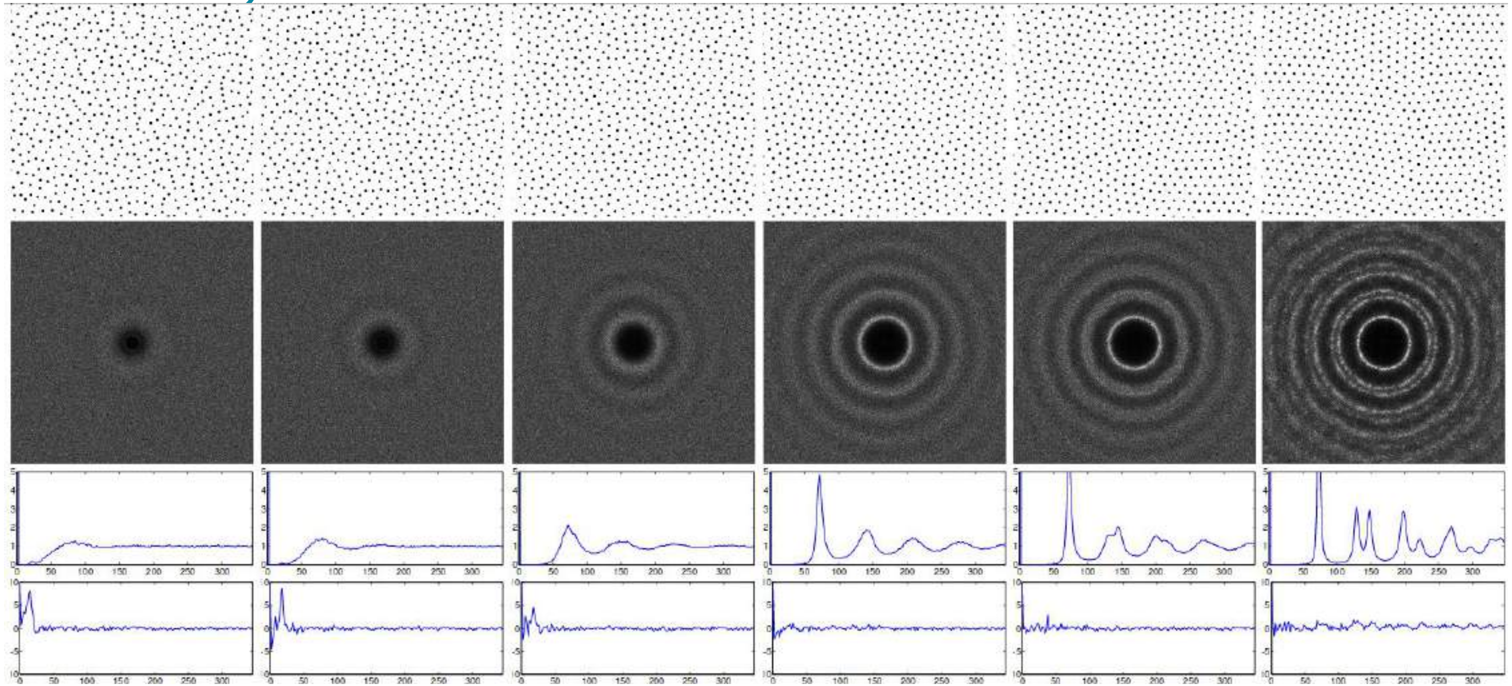
$$\sum_j \nabla W_{ij} \Rightarrow 0 \qquad\qquad \rho_i \Rightarrow \rho_0$$

$$\rho_i = \rho_0 + \lambda \quad \cdots \quad \rho_i = \rho_0 + 0$$

where λ is some user defined **density difference** controlling rate of convergence. *Rest density reset in each timestep as:*

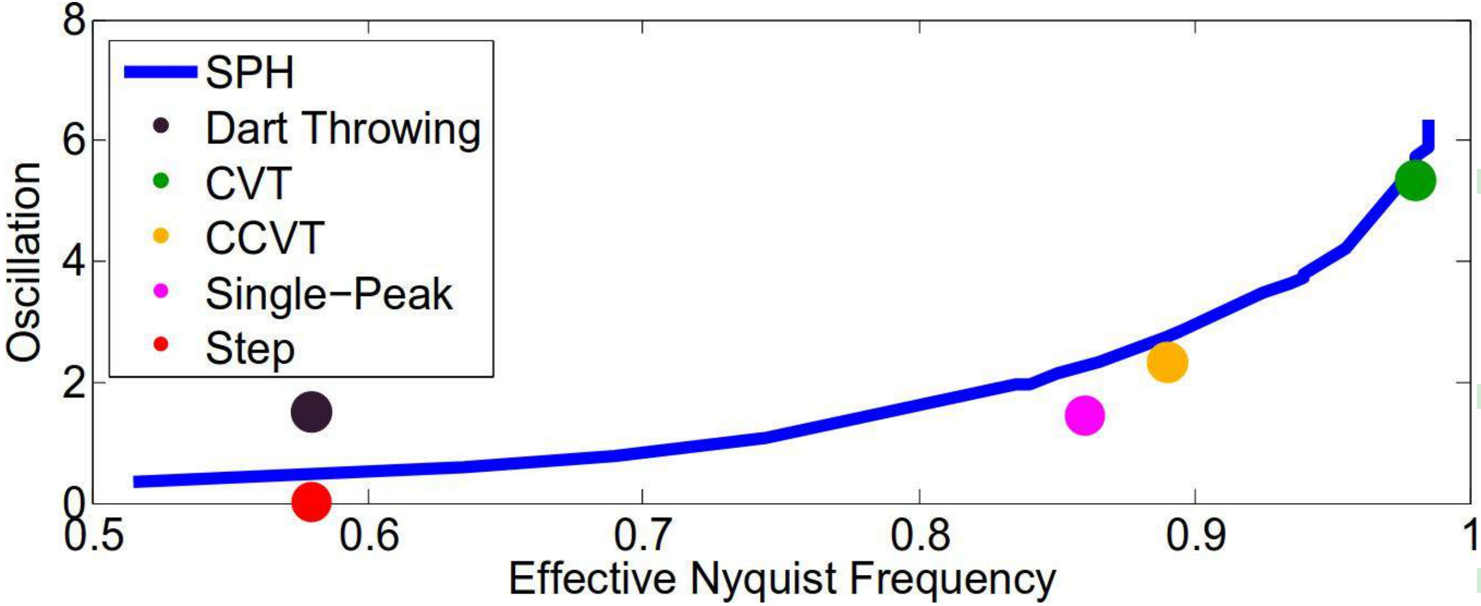
$$\rho_0 = \frac{1}{N} \sum_i (\rho_i - \lambda)$$

Family of results



$\lambda =$ 2 4 8 25 100 500

Effective Tradeoff



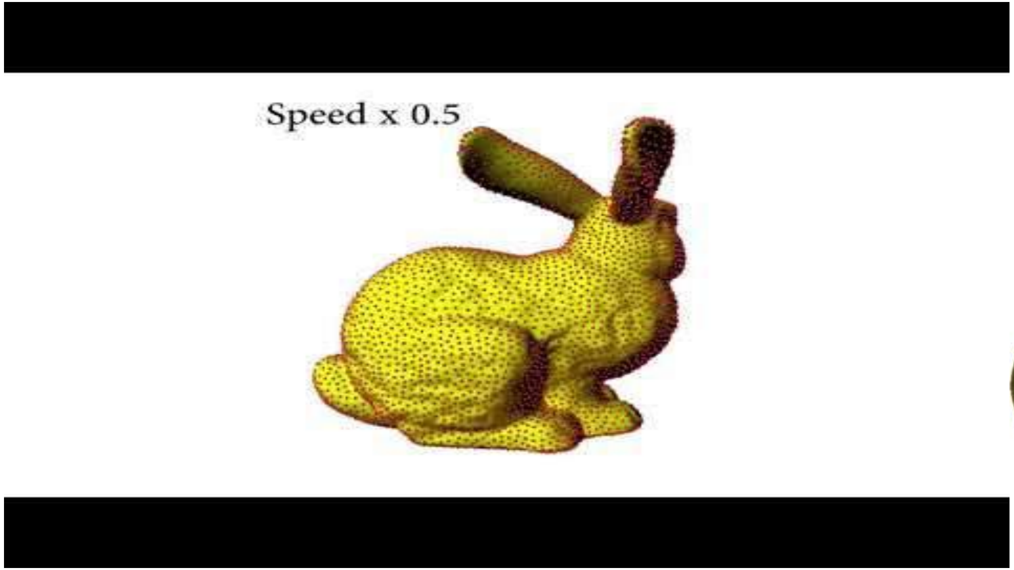
Further details explained in [9]

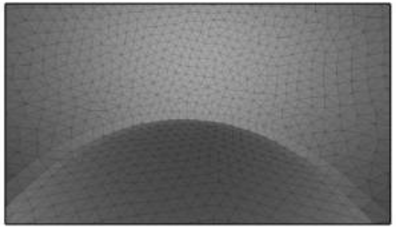
- Surface relaxation force
- Surface tension force
- Implementation details

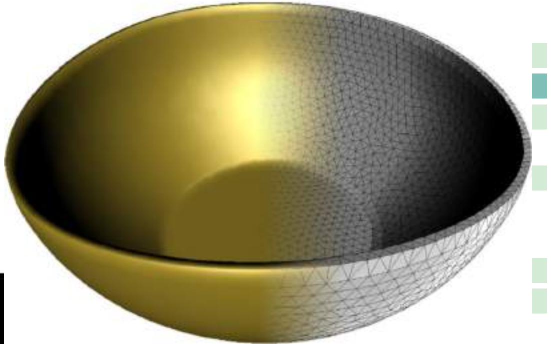
Applications of SPH Sampling

Surface Sampling

Sampling with geodesic distance metric on surface.







Adaptive sampling with a size function

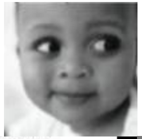
Modify distance based on some continuous function of space s:

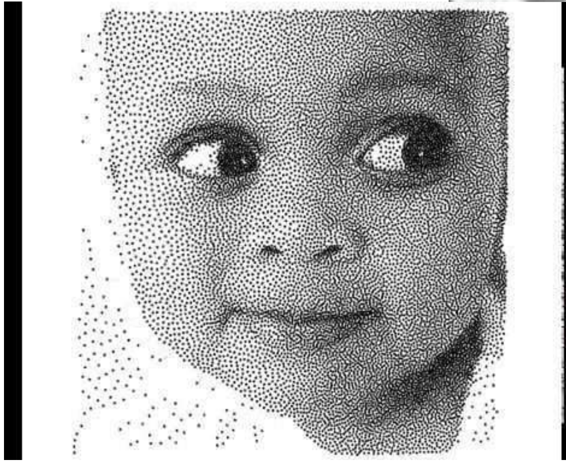
$$\tilde{s}\left(x_i, x_j\right) = \frac{2(x_i - x_j)}{s(x_i) + s(x_j)}$$

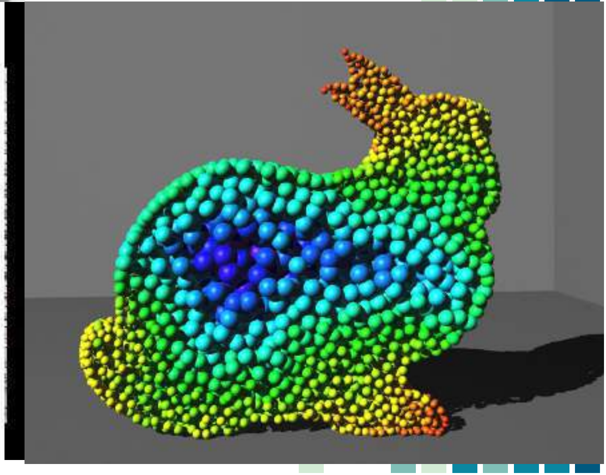
so kernel function becomes

$$W_{ij} = W(\tilde{s}(x_i, x_j), h)$$

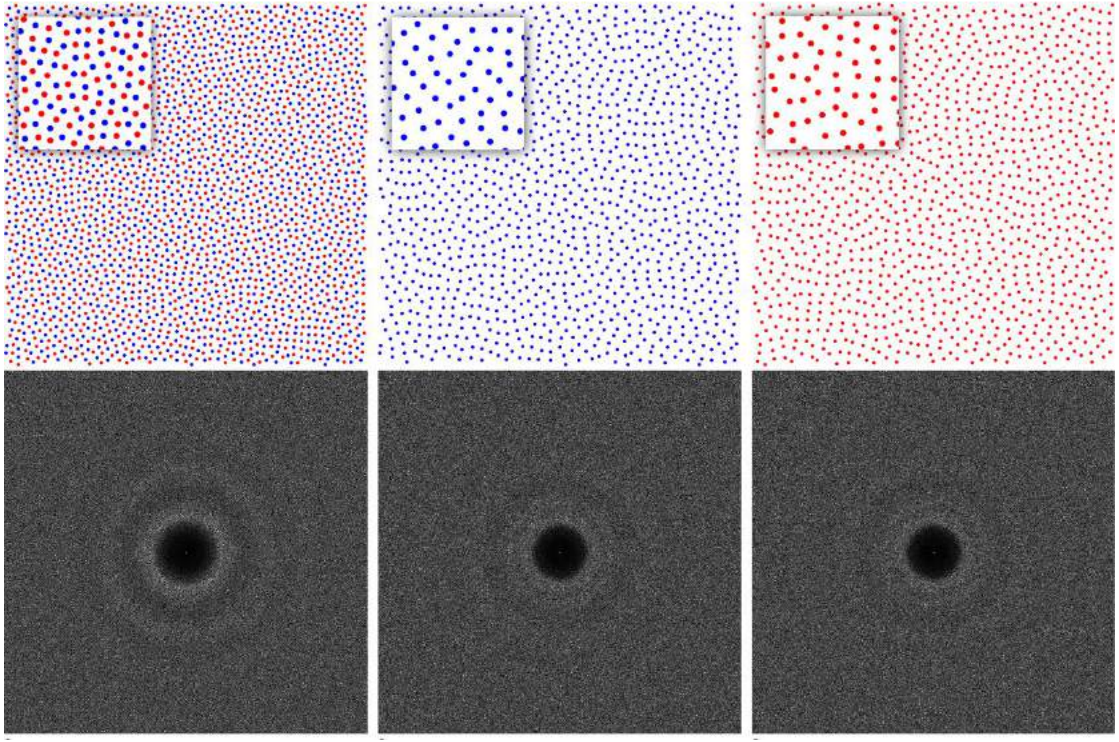
allows adaptive sampling.



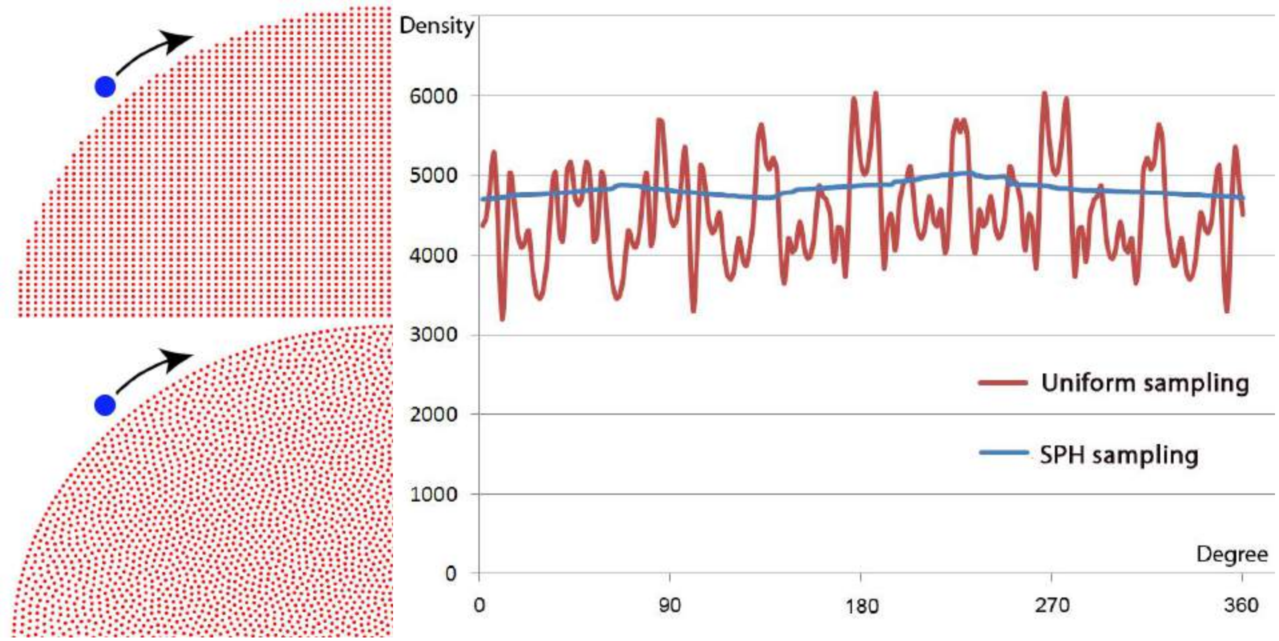




Multiclass Noise



Boundary sampling



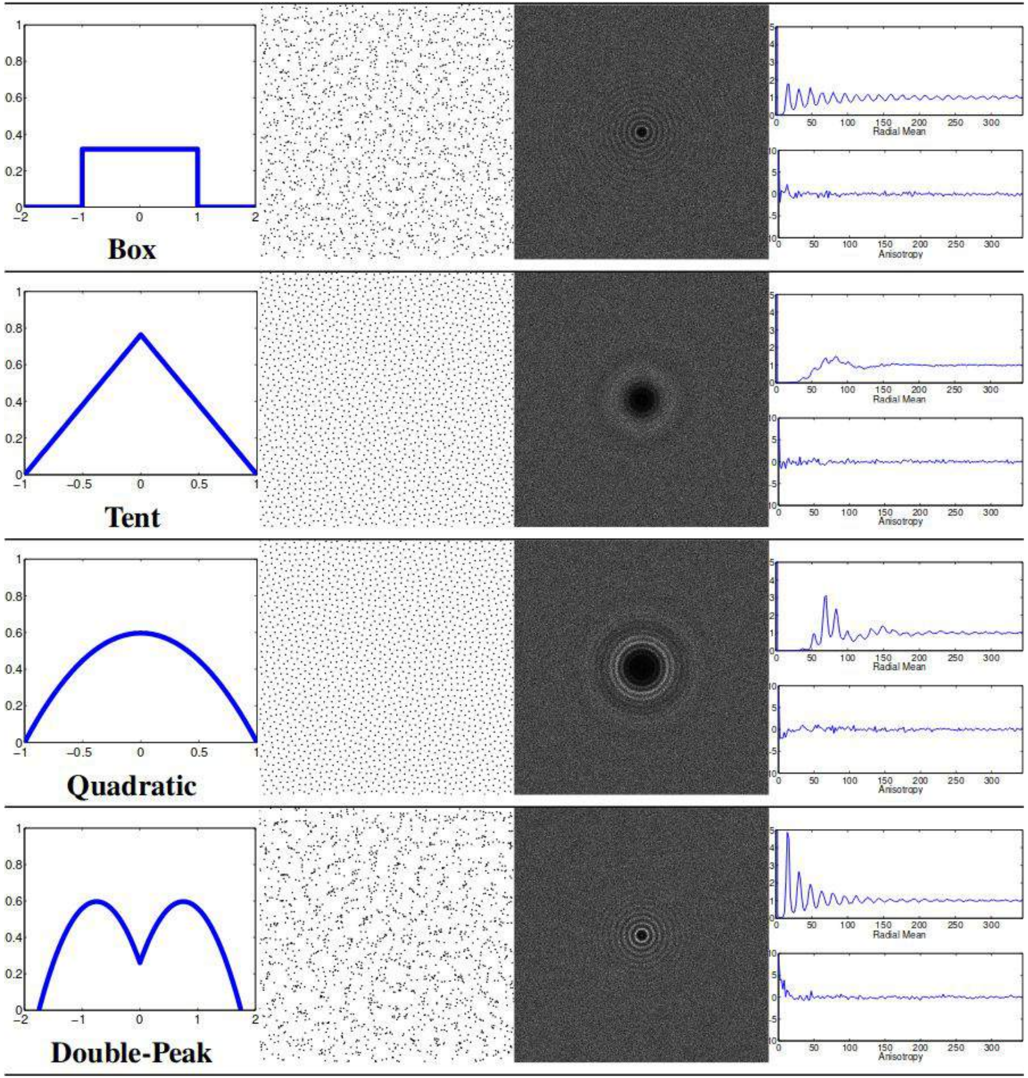
11. Jiang M., **Energy-based dissolution simulation using SPH sampling**, http://dx.doi.org/10.1002/cav.1798

Concluding remarks and Open Questions

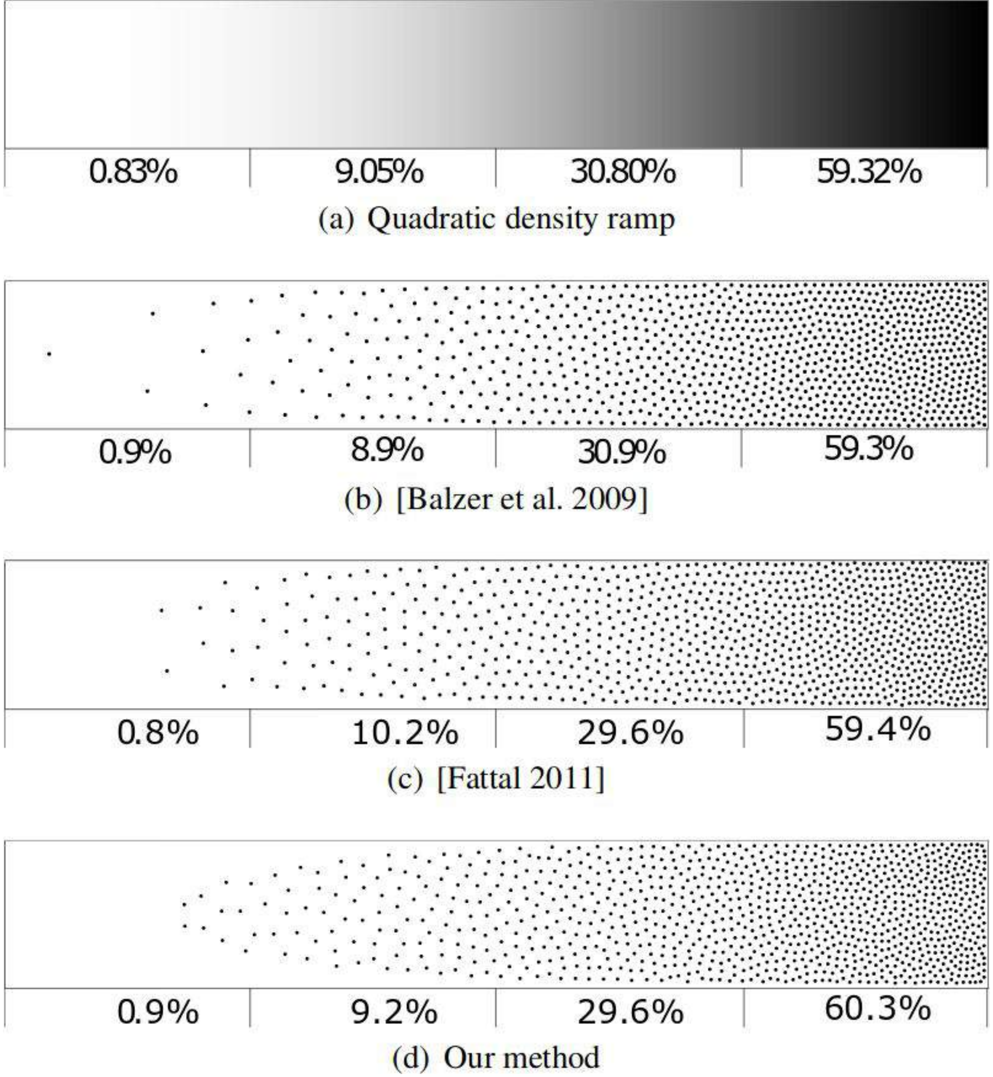
Properties of SPH Sampling

- **Controllable tradeoff between noise profiles**
- Adaptive sampling with size function
- Implicit surface / volumetric sampling properties
- Multi-class blue noise sets
- *Difficult to implement*
- *Comparatively slow*

Kernels functions and noise profile



Sampling in sparse regions



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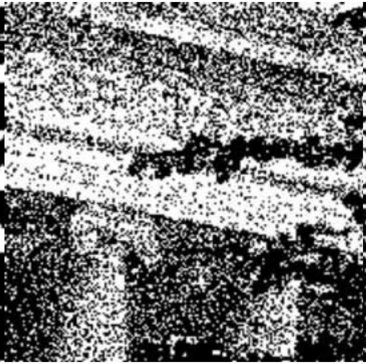
Halftoning Comparison (from [1])

Threshold



White Noise

Supercell

Adaptive Tessellation

Error Diffusion